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Part III

The Text re-arranged

THE
BAKHSHĀLĪ MANUSCRIPT

A Study in Mediæval Mathematics

BY

G. R. KAYE



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PART III.

1.—The Text Re-arranged.

Quotations in the text are distinguished by daggers† †, and abbreviations by ° superscribed. Asterisks attached to numbers denote change-ratios (See § 103). In the foot-notes angular crotchets < > indicate that the portion enclosed formed part of the argument or was implied in the original text, but is now missing.

On pp. 13 and 14 of Part I are tables equating the Bodleian Library order with the revised arrangements.

The notes attached to the revised arrangement are very crude and are presented with considerable diffidence; but they are the result of much labour and will possibly save the student of the MS a good deal of spade work.

G. R. K.

Owing to Mr. Kaye's unfortunate death, the last proofs of this part have been prepared for the press by Mr. K. N. Dikshit, Deputy Director General of Archaeology for Exploration, who has also made a few emendations.

A 1.

. . . yatra y.g bhāgam chaiva kārayet kshetra vaipulya 40^a recto.
 . . prishthā śata-dvayam chaiva uchare śatam ekataḥ vaipulyād vi
 . . śa dvādaśa nṛi śakas tathā | sapta pañcha bhavet chānam bhakti sthāne 39^a recto.
 . . r dhā sapta pañchānām tri-dvi meka ϕ prakalpitam | tasya
 vāhasya kim ka tatrā mama | kshetrasya.
 sthāpanam kṛiyate |

kshetram 100

15	12	10	7	5
----	----	----	---	---

karanam | kshetra . . .
 300 38^b recto.
 vaipulyād yogam . . .
 esha shaṭ 39^b recto.

. chaiva tat phal guṇitā jātā 40^a verso.
 6210 | esha vāhasya kāṇḍa pramānam . . . śake mūlyam kartavyam |
 adha chchhedam chatus sashṭhi la | sutha dvi triniśabhi maṇḍalakai 39^a verso.
 tallika esa chchhedam bhavati . . . yathe chchh . . . kāryā | sutha ṭala
 kṛiyā udāharanam . . . ṭalasya . . . mekam ta dvā sashṭhi śatānām
 daśādhikānām kim mulyam : ṭalla . . . tale a° 38^b verso.

1 . . . rū	1 mūlye	6210 maṇale abhim	pha . . .
1			

[38-40.] Plates xxvi and xxvii exhibit some sixteen fragments all placed out of order. Some of these have now been pieced together. (See the illustration facing page 4.)

This grouping is not final because some of the fragments consist of portions of two or more leaves stuck together, and until these are separated no exact order can be achieved.

We should naturally expect the first leaves of the manuscript to be comparatively more damaged than those in the middle of the book, and the 'find order' and the writing indicate that these fragments are probably portions of early leaves; but neither of these criteria is rigorous and it is quite possible that we have placed the fragments in their wrong places.

[38-40 recto.] These fragments appear to relate to a geometrical problem concerning an area whose width (*vaipulya*) is increased.

[38-40 verso.] A fragment of a problem connected with the area of a circle or the surface (*ṭala*) of a sphere. The phrase *esha vāhasya kīṇḍa pramānam* ought to be illuminating but is not. The change-ratio 64 is possibly connected with a "square measure." See Part I §108(b). The number 6210 = 3³·230 is said to be the product of certain quantities.

A 2.

. ksh daśa | chatur-daśa tritī- 39^b recto
yasya chaturthasya bhāgās tasyaiva pañchama . . . 40^a recto
. . . bhāgā vimśas cha dasagunā | saptama ksh . . . jñā 38^a recto
yañ śatam | sarve miśrāpi dṛishṭham cha śatāni . . . 40^b recto
. . . dhanam 1 10 || esha ekaika bhāgā guṇitā jā 39^b verso
60 | 180 | 200 | 300 | evaṁ dhanam 1200 pratyaya trairāsikena . 00 40^a verso
. dhanam 1200

.	pha° 144	38 ^a verso
.	pha° 16.	39 ^d verso
.	pha° 180	40 ^b verso
20	pha° 200	

A 2. [38-40.] The writing on the two sides differs (*recto* α_1 , *verso* α_2) and there are other indications that the fragments consist of portions of two leaves at least.

A 3.

bdhāmbupayaso ghataḥ eka miśrikṛit 40^d recto.
karaṇam | havya tulyam vinikshipyaḥ 40^a recto.

4	5	6
4	5	6
4	5	6

kuru prakshepakam tata praksh

A 3. [38-40 recto.] See the plate facing page 4. The meaning is not clear, but $x(x+y+z)=60$
 $y(x+y+z)=75$
 $z(x+y+z)=90$
whence $(x+y+z)^2=225$ and $x+y+z=15$. The answers are $x=4$, $y=5$, $z=6$.
The writing is classed as α_2 .

A 3—contd.

sthāpya

4 pa 15	5 am 15	6 15
4 15	5 15	6 15
4 15	5 15	6 15

39² recto.

. kṛiyate || chaturbhi pañchabhish shadbhi g . . prathama

rāśi yoga

60
15

 vartyaṁ 4 madhū ghaṭa . . . dvitīya pañktyā yoga

75
15

 38^a recto.

vartyaṁ

5
1

 pānīyaṁ || tritīya pañktya kṛiyate yogaṁ

90
15

 vartyaṁ jātāṁ

6
1

 payasāṁ

. kṛitvā . . . guṇetu || eko . . . kṛitāṁ 40^d verso.

śatatrayaṁ pañchabhi ṣ purushair labdham kiṁ ādyaṁ prathamam dhanam ||

. 120 || . 2257 . . ṇam t śeshe kshepa

16 anenātra bhāga

32
16

 labdha

2

 .

40
12

 pha 38^a verso.

. . labdher bhāg.

28
2

 jātā 14 labdha kshepaṁ dr̥i°60

prakshepa yukti 30 vibhaktam

1
30

 ṇitā jātā . . 14 | 18 38^a verso.

| 28 | evaṁ 60

A 4.

. dviguṇam cha tri-ūna cha tritīyasya dhanam bhavet 54^a verso.
 samyutam | eka-vimśatibhi ḥ krīto dīnāraistu rai ya
 tu dam sā prithag vachah ||
 karam || yasya padam na jāyate etat prathamasya
 dhanam |
 cha dattavān hastag yeshām | 0 | 2 54^a verso.
 | 2+

54^a recto.

dhanam . . .	1	2	4	8
	1	1	1	1

yātā | tayor yogaviyo kṛtām rāshayah

2 ¹	1	2+	9+	dī ^o 82
1	1	1	1	
1	2	4	8	
1	1	1	1	

. bhājyā hitveti | tatra uttara rāśi uttaram riṇam jātam

(b) sūtram || (c) jātam 76 esha prathamasya 54^b recto.

[54.] Folio 54 possibly consists of two leaves, or rather fragments of them, for there are ten pieces. The writing on the two sides differs—that on 54^a recto may be classed as α_1 and that on the left side as α_2 and in this respect the leaf resembles fol. 35^b. There is a characteristic *ye* at the bottom of 54^b verso which is also found on 29^b recto et verso.

[54^a verso.] Seems to contain portions of a *sūtra*, an example and solution. The phrase *dviguṇam cha tri-ūna* seems to be referred to on fol. 35^a recto but there we have *tryūna* with a particularly noteworthy conjunct *tryū* (see table IV, 5 part ii). The term *hastag(ataṇi)* on 54^b recto (not necessarily connected with 54^a) occurs only once more on fol. 1^a recto.

[54^a recto.] The phrase *tayor yogaviyo* also occurs on fol. 35^b verso.

A 5.

. kasmāt kāraṇā | tayor yogaviyogasy āviyogas 35^b recto.

bhājitā puruṣa 15 anena bhaktvā dhanam

9
15

 padvaya .

sahitam ||

. mūleṇa

1
1
2

 eta dviguṇam

3

 dviyuta yasya 35^b verso.

dhanam | tadeva svārdham

3

 asyārdham

1
1
2

 yutam nyāsa

5. [35^b.] The writing is different on the two sides (α_1 and α_2) and possibly the fragment is a portion of two leaves stuck together. The phrase *bhājitā puruṣa* . . . occurs on 51^b recto.

A 6.

bhājita hitvā | tatrottarā 1 | 1 | yutam $\begin{array}{|c|} \hline 2 \\ \hline \end{array}$ | $\begin{array}{|c|c|c|} \hline 1 & 3 & 3 \\ \hline 4 & 2 & 1 \\ \hline \end{array}$ 51^b recto.

9 | . eshā ϕ . itha bhājita | purushah $\begin{array}{|c|} \hline 1 \\ \hline 4 \\ \hline \end{array}$ | $\begin{array}{|c|} \hline 3 \\ \hline 2 \\ \hline \end{array}$ | $\begin{array}{|c|} \hline 3 \\ \hline 1 \\ \hline \end{array}$ | eshām sadṛiṣe 35^a recto.

$\begin{array}{|c|} \hline 4 \\ \hline 19 \\ \hline \end{array}$ dhanam $\begin{array}{|c|} \hline 19 \\ \hline 1 \\ \hline \end{array}$ anena guṇitam jātām $\begin{array}{|c|} \hline 4 \\ \hline \end{array}$ esha prathamasya dhanam

. . . . dviguṇam $\begin{array}{|c|} \hline 12 \\ \hline \end{array}$ dvi-yutam | 14 | eta dvitīyasya

guṇam $\begin{array}{|c|} \hline 21 \\ \hline \end{array}$ dvi-guṇam $\begin{array}{|c|} \hline 42 \\ \hline \end{array}$ try-ūṇam $\begin{array}{|c|} \hline 39 \\ \hline \end{array}$ eshaḥ nyāsaḥ

pratya daśam agravṛindānām chatur-daśa ekonachatvāriṃśa | tat

pād-ārdha tri-bhāgā

$\begin{array}{|c|} \hline 4 \\ \hline \end{array}$ | $\begin{array}{|c|} \hline 1 \\ \hline 4 \\ \hline \end{array}$ | pha° 4 | .evam di° 21 | esha prashṇa etair . . .

[51^b and 35^a recto.] The position is uncertain but the writing is of the α_2 style and there are slight indications of connexion with folio 54. Fol. 35^b is in α_1 writing. See the plate facing page 4. (Read 51 recto B, not verso.)

The fragmentary contents are not clear. We have $1+1=2$; $\frac{1}{2}+\frac{1}{2}+3=\frac{13}{2}$; $\frac{19}{10/4}=4$ and $\frac{2.3.(12+2)}{2}-3=39$

Apparently a fragment of the *sūtra* on which the solution depends is preserved on fol. 54^a verso, but the evidence, consisting of the phrase *dviguṇam cha tri-ūna*, is slender.

udā || | 6 | yoga 111 śeshā ϕ 51^b verso.

purusha bhājita purushah $\begin{array}{|c|} \hline 1 \\ \hline 4 \\ \hline \end{array}$ | $\begin{array}{|c|} \hline 1 \\ \hline 6 \\ \hline \end{array}$ | $\begin{array}{|c|} \hline 1 \\ \hline 5 \\ \hline \end{array}$ | eshām sadṛiṣhe yutīm kṛitvā yutā $\begin{array}{|c|} \hline 37 \\ \hline 60 \\ \hline \end{array}$ | 35^a verso.

bhājita $\begin{array}{|c|} \hline 60 \\ \hline 37 \\ \hline \end{array}$ | esha gavāśva mahishī pratyaika śāleshu bhāga . .

$\begin{array}{ c } \hline 1 \\ \hline 1 \\ \hline \end{array}$ śā°	$\begin{array}{ c } \hline 180 \\ \hline 1 \\ \hline \end{array}$ gā°	$\begin{array}{ c } \hline 1 \\ \hline 4 \\ \hline \end{array}$	phalaṁ 45	
$\begin{array}{ c } \hline 1 \\ \hline 1 \\ \hline \end{array}$ śā°	$\begin{array}{ c } \hline 180 \\ \hline 1 \\ \hline \end{array}$ asvā°	$\begin{array}{ c } \hline 1 \\ \hline 6 \\ \hline \end{array}$	phalaṁ 30 4+	26
$\begin{array}{ c } \hline 1 \\ \hline 1 \\ \hline \end{array}$ śālā	$\begin{array}{ c } \hline 180 \\ \hline 1 \\ \hline \end{array}$ mahi°	$\begin{array}{ c } \hline 1 \\ \hline 5 \\ \hline \end{array}$	phalaṁ 36 5+	9

[51^b & 35^a verso.] The writing is of the α_2 class. The 'find order' of folio 51 is 37 while that of 35 is not known. The position is very uncertain. What remains of the problem is

$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{3}{2}$ 111 $\times \frac{2}{3}=180$
 1 enclosure : 180 cows : $\frac{1}{2}$: 45 ? subtract 6 = 39.
 1 " : 180 horses : $\frac{1}{2}$: 30 subtract 4 = 26.
 1 " : 180 buffaloes : $\frac{1}{2}$: 36 subtract 5 = 29 ?

1	1	1	1	
1	1	1	1	

. . . . | 1 | 2 | 3 | eshām yuta

6
1

 | 48 | śeshā φ

purusha sa 4 || anena bhājitā-r-labdhāsya bhavati | 12 | 13
| 14 | 15 | ekatram 54 ||

udā° || kaśchid rājā dade dānam sapta - pañchāśakam budha |

. pañchā pravakshyāmy = anupūrvaśah

dvi-guṇa dvi-guṇam chaiva rūpa rūpottare

. . . prathame prāptam kim prāptam apare jane ||

0	1	2							
1	1	1							

						dri	329
							1
1	3	9	27	81			

karaṇam | uttar tatrottara rāśinām yoga 87 esha dhanā

drishyā śodhanīyā jātā 242 | purusha | 1 | 3* | 9 |

27 | 81 | yoga 121 anena jātā

2

 esha dvau

prathamasya dhanam ||

2 | 6 | 18 | 54 | 162 | uttara rāśi saṁyutam jātām

2	15	48	147	444	eshām
1	2	2	2	2	

7. [51^a.] Either there are two leaves stuck together here or there is some over-lapping. The writing on both sides is α_2 . The find order is 37.

[51^a recto.] i. There is not enough material for reconstruction but $x+(x+1)+(x+2)+(x+3)=54$ therefore $4x=54-6$ and $x=12$ is indicated.

ii. A certain Rāja makes presents to 57 wise men, etc. See 52 recto.

[51^a verso.] This apparently does not connect up with the other side. It exhibits the solution of an example which may be expressed by

$$t_1 + 3t_1 + 3^2t_1 + 3^3t_1 + 3^4t_1 + \dots = 329$$

Set $t_1=2$ then the first series becomes 242 and the second 87 and the combined series is

$$2 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = 329. \text{ See also page 47.}$$

* Omitted in the MS.

A 8.

. 57 . . . tvedaṁ jātāṁ 3 anena chālīmśa guṇaye 52 recto.
jātā 120 vaṁ śūrānāṁ ||

pratyaya trai-rāsikena	1 1	vaṁ	120 1	1 10	12 1	...
	1 1	vaṁ	120 1	1 8	15 1	...
	1 1	vaṁ	120 1	1 4	30 1	...

udā° || dhanā sva-m-ardho saṁśoddhya . . . chottariyakam |
tat seshā pañchamo bhāgo . . . śata dvayaṁ |
aśītyādhikam dhanam chaiva kiṁ ādyaṁ prathamam dhanam ||

. asya dvayānām śatānām pāda . . . rdham 52 verso.
śataṁ bhavati 150 atrāpi pañcha bhāga 30 || evam

1 2	1 4 1 2 ⁺	1 5 1 4 ⁺ 1 2 ⁺	pha° piṇḍa 280
--------	-------------------------------	--	----------------

pañchamī jātī karaṇam kṛita . . . 280 | amśa yuti | 28 | bhaktam | 40 |
40 | 28 |
dhanu 280 guṇitam jātām 400 esha phalam bhavati ||

[52 recto.] i. The writing on both sides is α_2 and exhibits examples of the 'sickle-shaped' medial \bar{i} and \bar{I} . The 'find order' is 57. It is possible that 52 recto gives parts of the solution of the example on 51 recto which would make that page the reverse, but I doubt the connexion. What is left of the solution means

$$x(\frac{1}{10} + \frac{1}{8} + \frac{1}{4}) = 57 \quad \text{or} \quad x\frac{10}{40} = 57 \quad \text{and} \quad x = 120. \quad \text{A proof by the 'rule of three'}$$

$$1 : 120 :: \frac{1}{10} : 12$$

$$1 : 120 :: \frac{1}{8} : 15$$

$$1 : 120 :: \frac{1}{4} : 30$$

ii. The example, which is continued on 52 verso, may be expressed by $x(1 - \frac{1}{2})(1 - \frac{1}{4})(1 - \frac{1}{8}) = x - 280$ whence $x = \frac{280}{\frac{25}{32}} = 400$. A proof follows: $\frac{400}{2} = 200$ and $400 - 200 = 200$; $\frac{200}{4} = 50$ and $200 - 50 = 150$; $\frac{150}{8} = 30$ and $150 - 30 = 120$; and $400 - 120 = 280$.

$$\text{Again } \frac{1}{2} + \frac{1}{4} (1 - \frac{1}{2}) + \frac{1}{8} (1 - \frac{1}{4}) (1 - \frac{1}{8}) = \frac{1}{2} + \frac{1}{8} + \frac{1}{40} = \frac{35}{40} \text{ and } 280 \times \frac{40}{35} = 400.$$

A 9.

. vinirdiset || 29^d recto.
 udā^c || dhana
 ādya dvitiya yonmiśraṁ dhanam tatra ttrayodashaḥ
 dvitiya tṛitiya yonmi chaturdaśa
 ādya tṛitiya yonmiśraṁ dhanam pañchadaśa smṛitaḥ
 ekaikasya dhanam chchhiche katthyatām mamah

13	14	15
1	1	1

 prathamam yasya tatrechhā pañchah | 5 | tat prathama . . . 29^b recto.
 . . . 13 14 | 15 | tadādiś† śodhayet kramāt† ādi eta

13	14	15
1	1	1

 chatur-daśabhi śodhya śesham | 6 | etat pañcha 29^c recto.
 dvitiya yonmiśraṁ dhanam 29^d verso.
 dvitiya tṛitiya yonmiśraṁ dhanam sapta-dasha smṛitaḥ
 tṛitiyaś chaturthayo
 chatuṣ pañchaka miśraṁ tu dhanam ekona-vimśati |
 prathama tatra cha
 ekaikasya dhanam kimssyād vechchhi

16	17	18	19	20
1	1	1	1	1

 29^b verso.
 karaṇam || ichchhā dani†śodhayet kramāt† tatrādi 16
 śud tṛitiyāyam śodhya 7 chaturthāyam śodhya 12 pañ 29^c verso.

9. [29.] Folio 29 consists of six fragments, of which only the four larger ones need be considered at present. The correct order is *d, b, c*. Fragment *b* fits under *d* and *c* under *b* while *a* goes with folio 27. See the plate facing page 4.

[29 *d, b, c recto.*] The problem and its solution here partly preserved may be represented by $x_1 + x_2 = 13$, $x_2 + x_3 = 14$, $x_3 + x_1 = 15$. If $x_1 = 5$ then $x_2 = 8$, $x_3 = 6$ and $x_3 + x_1 = 11$ and the correct values are found from $x_1 = 5 + \frac{15-11}{2} = 7$, $x_2 = 13 - 7 = 6$, $x_3 = 16 - 7 = 9$. The phrase "śodhayet kramāt" recurs in the next example and is a quotation from a lost *sūtra*.

[29 *d, b, c verso.*] The example here given (continued on folio 27 *verso*) is formulated with exactly the same phraseology as the previous one. It may be represented by

$x_1 + x_2 = 16$, $x_2 + x_3 = 17$, $x_3 + x_4 = 18$, $x_4 + x_5 = 19$, $x_5 + x_1 = 20$. If $x_1 = 10$, $x_2 = 6$, $x_3 = 11$, $x_4 = 7$, $x_5 = 12$ and $x_5 + x_1 = 22$

Therefore the correct value of x_1 is $10 + \frac{20-22}{2} = 9$, $x_2 = 7$, etc.

The phrases *ichchhā* . . . and *śodhayet kramāt* are quotations from a lost *sūtra*.

A 10.

. masya dhanam | esham anukkrameṇa 27 verso.
pūrvokt

9 pra°	7 dvi°	10 tṛi°	8 cha°	11 pañ°
7 dvi°	10 tṛi°	8 cha°	11 pañ°	9 pra°

yutam jātam pratyak . . . 16 | 17 | 18 | 19 | 20

. . . evam sarvatra kārayet ||

29^a verso.

karaṇam | †prithak rūpam vinikshipya† | prithak rūpam kshiptam jātam . 27 recto.

†. . . bhyāso† tatra guṇa

3	4
---	---

 abhyāsam

12

 † rūpahīnam† 1 .

. . abhyāsā chatuṣ pañchakā | atra kshiptam jātam

15	16
----	----

eśa triguṇ tā mūla . . ni chatuṣ pañcha

5	4
---	---

 esha .

.

sūtram || guṇau ka dhanam ||

29^a recto.

guṇ ābhyāso rūpa hīnam labdham rū

A 10. [27 verso] gives the answer of the problem given on fol. 29 verso, namely $x_1=9$, $x_2=7$, $x_3=10$, $x_4=8$, $x_5=11$, and the sums of the pairs are 16, 17, 18, 19, 20. (For general discussion see § 78, Part I.)

[27 recto.] Solution of a lost problem which may have been $xy-3x-4y+1=0$ of which solutions are: $x = \frac{3y-1}{4}+4=15$, $y=3+1=4$; $x=4+1=5$, $y = \frac{3x+1}{4}+3=16$. The quotations are from a *sūtra* very much like the one that follows.

The phrase *prithak rūpam vinikshipya* 'having added unity in each case' appears to be a quotation from a lost *sūtra*.

[29a] is wrongly placed on plate XX. It should come directly under 27, for of the letters —*evam sarvatra kārayet*, the top portions are on 27 verso and the bottom on 29a recto.

The writing is classed as α_2 .

A 11.

i. bhyasa

..
1

 rū chaturguṇam pañchaguṇam hastagatam 1 recto.

dhanam ja pañchaguṇam 25 ||

navama sūtram 9

ii. sūtram || guṇau prithag rūpayutau yāchanā yukti samguṇāḥ ||

guṇanena guṇe . . rūpahīnena bhājītau |

viparīta yāchanā kshiptau guṇaśāster ayam vidhiḥ ||

evam sūtram || dvitīya patre vivaritāsti ||

daśama sūtram 10 ||

iii. sūtram || amśam viśoddhya chchedebhya kuryātat parivartanam ||

. . . sāsyam tata projjhya dhanānviśa vinirdiśet ||

iv. udā° || pañchānām vaṇijā madhye maṇi vikriyate kilah

tatroktā maṇi vikrīta maṇi mūlyam kiyad bhavet

. dam

ardha tṛi-bhāga pādānśam pañcha-bhāga śodamśa cha

. †tato projjhyah† sadriśam kriyate jātā 1 verso.

120	90	80	75	72
60	60	60	60	60

tatra projjhyah† jātam 120 | 90 | 80 | 75 | 72

eshām yoga kṛite jāta 437 ato . . . śesham 377 eśa maṇi mūlyam |

A 11. [1 recto.] The position is uncertain. The 'find order' (33) places this leaf next to the fragments of folios 27, 29, 38, 39, 40. The writing is α_2 (there is a 'sickle' α). The numbered *sūtras* seem to place the leaf fairly early but they are not a very safe criterion. Note (ii) below seems to connect folios 1 and 27.

(i) Nothing intelligible. It ends the earliest numbered *sūtra* preserved.

(ii) I have not yet made out the meaning of this *sūtra*. Compare the opening phrase with the quotation on 27 recto. The metre is irregular. The reference to the second leaf is possibly to folio 27.

(iii) The *sūtra* means change $\frac{a}{b}$ to $\frac{b}{b-a}$ and quotations from it are given on folios 1 verso and 2 verso.

(iv) The example is solved on 1 verso and 2 recto and appears to have been somewhat as follows:

The combined capitals of five merchants less one-half of that of the first, one-third that of the second, one-fourth that of the third, one-fifth that of the fourth, or one-sixth that of the fifth is equal to the cost of a jewel. Find the cost of the jewel and the capital of each merchant.

A 11. [1 verso.] This appears to give part of the solution and proofs of the question on 1 recto. Since $\sum x - \frac{1}{2}x_1 = \sum x - \frac{1}{3}x_2 = \sum x - \frac{1}{4}x_3 = \sum x - \frac{1}{5}x_4 = \sum x - \frac{1}{6}x_5 = c$, we have $\frac{1}{2}x_1 = \frac{1}{3}x_2 = \frac{1}{4}x_3 = \frac{1}{5}x_4 = \frac{1}{6}x_5 = k >$ whence $\sum x = \frac{120+90+80+75+72}{60} = \frac{437k}{60} = \frac{437c}{437-60}$.

If $k=60$ then $c=377$ and $x_1=120$, $x_2=90$, $x_3=80$, $x_4=75$, $x_5=72$.

Then follows a 'proof' which may be expressed by

$$\begin{aligned} 90+80+75+72 &= 317 \text{ and } 317 + \frac{120}{2} = 377 \\ 120+80+75+72 &= 347 \text{ ,, } 347 + \frac{90}{3} = 377 \\ 120+90+75+72 &= 367 \text{ ,, } 367 + \frac{80}{4} = 377 \\ 120+90+80+72 &= 362 \text{ ,, } 362 + \frac{75}{5} = 377 \\ < 120+90+80+75 &= 365 \text{ ,, } 365 + \frac{72}{6} = 377 > \end{aligned}$$

† Compare with *sūtra* 11 on 1 recto.

A 11—contd.

chaturthām saṅka sarvasvam ||

prathamasya saṅka ardham . . . 90 | 80 | 75 | 72 chaturthām yoga

317 prathamārdheṇa sashṭibhir yutam 377 eśa prathamasya dhanam

prathama dhanam | tṛtīya chaturtha pañchamasya dhanam sarvasvam 347

dvitīyā tṛi-bhāgam 30 eśa yutam 377 eśa dvitīyasya dhanam bhavati ||

puna prathama dvitīya chaturtha pañchama . . sarvasvam 357 tṛtīyasya

pādam 20 eśa yutam 377 eśa tṛtīyasya dhanam bhavati ||

punar api prathama dvitīya tṛtīya pañchamasya 362 chaturthasya pañcho-

bhāga 15 eśa yutam 377 eśa chaturthasya dhanam bhavati || . . .

A 12.

i. Sya dhanam bhavati ||

atha pratha tyamśashtī śesham 377 ||

2 recto.

atha dvitīyasya	120 30 80 75 72	evam 377 dvitīyasya bhavati
-----------------	-----------------------------	-----------------------------

atha tṛtīyasya kṛiyate	120 90 20 75 72	evam 377 tṛtīyasya dhanam bhavati
------------------------	-----------------------------	-----------------------------------

chaturthasya kṛiyate	120 90 80 15 72	evam 377 chaturthasya dhanam bhavati
----------------------	-----------------------------	--------------------------------------

pañchamasya kṛiyate	120 90 80 75 12	evam pañchamasya 377
---------------------	-----------------------------	----------------------

A 12. [2 recto.] i. This appears to be another 'verification' of the example on 1 recto et verso ; and means

$$<120+90+80+75+72=377>$$

$$120+\frac{90}{2}+80+\frac{75}{2}+72=377$$

$$120+90+\frac{80}{2}+\frac{75}{2}+72=377$$

$$120+90+80+\frac{75}{2}+72=377$$

$$120+90+80+75+\frac{72}{2}=377 \text{ and 'this is the measure of the price of the jewel.'}$$

A 12—contd.

esha maṇi mūlyam pra

ii. udā° || anyonya vidita vibhavam vaṇikdvayam |

trī dalam tatha . . .

7+	3+	5+
12	12	6
12	12	6

2 verso.

†amśām viśoddhya† visodhayet riṇam sthitam | esha . . . kṛiyate

19	7	11	†kuryātat parivartanam†	12	4	6	chchhede .
12	4	6		19	7	11	

. jātām asya	924	836	798	projjhya jātā	924	836	798
	1463	1463	1463				

eshām yutiṁ kṛiyate jātā | 2558 | chchheda projjhyam 1095 etan

maṇi mūlyam

A 12. ii. This is possibly the question solved on 2 verso.

[2 verso.] The general meaning is: since $x_2 + x_2 - (\frac{1}{2} + \frac{1}{2})x_1 = x_1 + x_3 - (\frac{1}{2} + \frac{1}{2})x_2 = x_2 + x_1 - (\frac{1}{2} + \frac{1}{2})x_3 = 0$, or $\Sigma x - (1 + \frac{1}{2})x_1 = \Sigma x - (1 + \frac{1}{2})x_2 = \Sigma x - (1 + \frac{1}{2})x_3 = c$, whence $\frac{1}{2}x_1 = \frac{1}{2}x_2 = \frac{1}{2}x_3 = \Sigma x - c$ and $\frac{\Sigma x}{\Sigma x - c} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$. Setting $\Sigma x - c = 1463$ we have $x_1 = 924$, $x_2 = 836$, $x_3 = 798$; $\Sigma x = 2558$ and $c = 1095$ 'which is the price of the jewel.'

I do not, however, understand the form of the first statement; but see fol. 65 verso where $\frac{83}{42}$ means $\frac{83 \cdot 1 + 40}{42}$.

†amśām viśoddhya and kuryātat parivartanam are quotations from sūtra 11 on fol. 1 recto.

A 13.

udā° || dvitīyasya hayān navah

3 verso.

ūshtrā dasa tritīyasya

. pradattam cha parasparam

prithag dhanam tu vaṇijām mūlyam vā prāṇinām prithak

yadi vaktum tato me chchhindhi saṁsayah ||

A 13. [3 verso.] Writing α_2 . Note the looped medial c in the penultimate line. Possibly a double leaf. 'Find order' 49. The position is determined only by the writing and the numbered sūtras on the reverse. Example: One possesses 7 horses ($\alpha^\circ = asva$), another 9 horses ($ha^\circ = haya$) and a third 10 camels ($\pi^\circ = ūshtra$). Each gives one of his animals to both the others (and then their possessions are of equal value). It is required to find the capital of each merchant or the price of each animal. If thou art able, solve me this riddle.

We have $(7-2)x_1 + x_2 + x_3 = (9-2)x_2 + x_3 + x_1 = (10-2)x_3 + x_1 + x_2 = c$ or $\Sigma x - (7-3)x_1 = \Sigma x - (9-3)x_2 = \Sigma x - (10-3)x_3$, whence $4x_1 = 6x_2 = 7x_3 = k$ and $\Sigma x = \frac{42+28+21}{168}k$. If $k = 168$ then $x_1 = \frac{168}{4} = 42$, $x_2 = \frac{168}{6} = 28$, $x_3 = \frac{168}{7} = 24$. Also $7x_1 = 294$, $9x_2 = 252$, $10x_3 = 240$ are the original capitals, and $c = 262$.

Mahāvira gives the following example.

Rule.—The number of gems to be given away is multiplied by the total number of men. This product is subtracted from the number for sale: the continued product of the remainders gives rise to the value of the jewel provided the remainder relating to it is given up.

Example.—The first man had 6 sapphires, the second had 7 emeralds and the third 8 diamonds. Each by giving to each the value of a single stone became equal (in wealth to the others). Answer. 20, 15, 12.

A 13—contd.

7 a°	9 ha°	ū° 10
1	1	1

vañijjakā 3 deyaṃ vaṇik piṇḍa hataṃ | piṇḍa 7 | 9 |
 10 | deyaṃ 3 śuddha śeṣhaṃ 4 | 6 | 7 tata śeṣhaṃ paraspara
 kṛitaṃ guṇita jātāṃ | 168 | 168 | 168 | svaśeṣhena tu vibhaktāṃ

168	168	168
4	6	7

 labdhaṃ 42 | 28 | 24 | eṣa pratyāika mūlyaṃ
 ekaikasya guṇitā jātāni asvai hayai ūṣṭreḥbhyah 294 | 252 |
 240 ekaikasya jātā 262 | 262 | 262 | eṣa sama dhanā

i. datvā ssamadhanā jātā prasta mūlyaṃ tad uchyatāṃ

3 recto.

4 ya°	5 go°	6 sa°
1	1	1

evaṃ prasta mūlyaṃ 2 | 3 | 6 dattais samadhanā jātā 17 | 17 | 17

trayodasama sūtraṃ 13

ii. sūtraṃ || ekayutānāṃ saṅkhyā dvi hīnā cha ||

evaṃ tāvat kāryaṃ yāvat puruṣai samā bhavati ||

saptama patre bhilikhita sthita

chatur-daśama sūtraṃ 14

iii. sūtraṃ || gatisyaiva viśeṣhaṃ cha vibhaktāṃ pūrva gaṃtunāḥ

tenaiva kālaṃ bhavati stha kena tu ||

iv. udā° || addhyardha yojana gate śata

.

A 13. [3 recto.] This is the reverse because sūtra 15 obviously begins a new section (B).

i. This appears to be a companion example to that on 3 verso. The abbreviations are possibly ya° for yava 'barley,' go° for godhūma 'wheat,' sā for sālī 'rice.' Here (4-2) x₁+x₂+x₃=(5-2) x₂+x₃+x₁=(6-2) x₃+x₁+x₂=c whence x₁=2x₂=3x₃ and x₁=6, x₂=3, x₃=2 and c=17.

ii. Not understood. The reference to the seventh leaf is now only tantalising. No recognisable quotations from the sūtra are preserved. The phrase tāvat yāvat 'so much as much' does not recur anywhere. In Bhāskara tīvat and yāvat (tā° and ya°) are used as algebraic quantities.

iii. The rule means $t = \frac{r_1 D}{r_1 - r_2}$, where r₁ and r₂ are rates of progress and D is a given time. (See § 83, Part I.) The rule is quoted on 4 recto where gatisyaiva viśeṣhaṃ cha and pūrva gata occur.

B 1.

i. sūtram || dviguṇaṃ prabhavaṃ śuddhā dviguṇaṃ niyathaṃ tathā

8 recto.

uttareṇa bhajech chhesham labdham rūpaṃ vinirdiśet ||

ii. udā° || vartate bhṛitakaḥ kaschi tatraiko daśa māśakam |

pratyaham kurute tatra karmam bhaṭṭika mānavah

dvitīyam kṛiyate karmam dvyādi tritayar uttaram |

padam tatra tu bhavati kena kālena sāsyatām ||

ā°	2	u°	3	pa°	0	prati°	10
	1		1		1		1

†dviguṇaṃ prabhavaṃ śuddhā† prabhavaṃ | 2 | dviguṇaṃ | 4 | niyata puna dvi

. | 16 | [uttarārdheṇa bhājayet] uttaram

i. sūtram || hayor vibhajya gantavyam ato bhāga . gantata

8 verso.

ekaś cha gamana jñeya yutās samguṇya

udā° || niyo rathośvair daśabhir yujyate haya pañchakam

gantavyam yojana śatam kim udbhavet

ha	10	haya lagna rathasya	5	gantavyo yojana	100
	1		1		1

†hayor vibhajya gantavyam† tatra hayā | 10 | gantavyam yo° | 100 | †ato

B 1. [8 recto.] The position of folios 8, 9 and 7 is very doubtful. They fit in nowhere perfectly. Their find orders are 48, 43 and 45; but 7 recto indicates that this find order is not of much value here. See the notes on fol. 7 verso. The writing is α4.

i. The rule is another variation of that given on 7 verso and means $t = \frac{2A-2a}{d} + 1$ where A is a fixed rate and $t A = ((t-1)\frac{d}{2} + a)t$.

ii. The example is $A=10$, $a=2$, $d=3$ whence $t = \frac{2 \cdot 10 - 2 \cdot 2}{3} + 1 = 6\frac{2}{3}$ and $s = \frac{1 \cdot 9 \cdot 0}{3} = 63\frac{1}{3}$.

The phrase *dviguṇaṃ prabhavaṃ śuddhā* is quoted from the *sūtra* above; while the phrase *uttarārdheṇa bhājayet* was wrongly quoted and was afterwards cancelled: Compare with the *uttarārdheṇa bhājitam* quoted on 7 verso.

B 1. [8 verso.] i. The *sūtra* is partially reconstructed from the quotations in the solution below.

ii. The example is: There are ten horses of which five are yoked at a time to the chariot. How many changes should there be in a journey of one hundred *yojanas* and how much will each horse do?

The solution is $\frac{1}{2} \frac{100}{5} = 10$ stages and $10 \times 5 = 50$

Proof. 5×100

Mahāvīra gives a similar example (vi, 158).

ravi-ratha-turagās sapta hi chatvāro'svā vahanti dhūryuktāḥ |

yojana-saptati-gatyaḥ ko vyūḍhaḥ ko chaturyogaḥ ||

'It is well known that the horses of the Sun's chariot are seven. Four horses are yoked at a time. They have to perform a journey of 70 *yojanas*. How many times are they unyoked and how many times yoked.'

Mahāvīra's solution is expressed thus:

The number of the total *yojanas* divided by the total number of horses gives the *yojanas* in turn. These *yojanas* multiplied by the optionally chosen number of horses to be yoked gives the measure of the distance to be travelled over by each horse.

That is $\frac{1}{2} \frac{70}{4} = 10$ is the length of each stage; and $10 \times 4 = 40$ gives the distance each horse works.

The solution is rather cryptic, but the interesting point is that the problem was a traditional one. Probably something of its original quality has been lost.

B 1—contd.

bhāga† *hṛite* labdha 10 tatra yuktāśva | 5 | etais saṁguṇya pariyoga jātām

. yojanānyaikośva rūḍha | pratyayaḥ pañchabhis śata saṁguṇya
jātām *kṛiyate* || yadi da yojana pañcha

B 2.

udā° || tat samāptam dvijanmabhi |

9 recto.

tat punas te samam bhaktvā daśa . . . samāptavān |

saṁkhyāya x kati māchakshu kati viprā x kati prashtam ||

ā°	1	u°	1	pa°	0	labdham	10
	1		1		1		1

karaṇam || ‡labdham dviguṇitam kritvā† tatra labdham | 10 | dviguṇam

| 20 | tathādvayūnam | 18 | ‡uttareṇa vibhājitam† atrottaram | 1 | anena

bhaktvā jātām tad esha rūpādhikam | 19 | ayam prashnā brāhmaṇā ekona-
viṁśati

sthāpa | ā° 1 | u° 1 | pa° 19 | rūpoṇā karaṇena phalam 190
| 1 |

9 verso

.
.. yo° 6 | śa° yo° 1 | yo° 70 | gantavyam |
| 1 | | 1 | | 1 |

B 2. [9 recto.] See the notes on fol. 7 verso. The writing is of the same style, α4. Possibly there are two leaves stuck together.

The example is a=1, d=1, A=10, and $10t = ((t-1)\frac{1}{2} + 1)t$ whence $t = \frac{2.10-2.1}{1} + 1 = 19$ and by the *rūpoṇa* method s=190.

Dr. Hoernle gave the following restoration :

"For a certain feast one Brāhman is invited on the first day, and on every succeeding day one more Brāhman is invited. For another feast 10 Brāhmans are invited on every day. In how many days will their numbers be equal; and how many Brāhmans were invited."

The use of the term *labdham* is here rather curious. The phrases *labdham dviguṇitam kritvā*, *tathādvayūnam*, *uttareṇa vibhājitam* and *rūpādhikam* are probably quotations from a *sūtra*.

B 2. [9 verso.] The example probably meant: 'A and B start for a place 70 *yojanas* distant. A travelled at the rate of 1 *yojana* a day and B at the rate of 6. At what point on his return journey did B meet A?'

Since $\frac{x}{1} = \frac{2.70-x}{6}$, where x is the distance traversed by A, we have $x = \frac{2.70}{6+1} = 20$ as given in the text, and since A travels at the rate of one *yojana* a day, this is also the time.

Proof by the 'rule of three' 1 day : 6 yo° :: 20 days : 120 yo°, and 70-20=50 and 70+50=120. Also 1 day : 170° :: 20 days : 20 yo°.

The abbreviation *sa°* may be for *sanairga* 'slow goer.'

B 2—contd.

a(la)bdhe samyoga

7
1

 vibhaktam

1
7

 gantavyena guṇitā jātān labdha

10

 dviguṇam

20

 eshālpasyaḥ ||

atha . . . ayam kālo jñeyaḥ anena kālenash shat yojanāni gantavyaṁ |

. . . bhyām ekayojanikasya samāgamo bhavati ||

tadyathā trai-rāśikena *pratyaya* | yady ekasya shat yojanā tadā vimśānām kiṁ

1	6	yo°	20	pha°	120
1	1		1		1

atha saptati śoddhya śeṣa atra ssaptati

70

 āgata pañchāśa

50

adhve

.

1	di°	1	yo°	20	di°	pha°	yo°	20
1		1		1				1

B 3.

i.

7 verso.

ā°	3	u°	4	pa°	0	nitya	datta	7
	1		1		1			1

†ādim viśoddhya† ādi

3

 niyataṁ

7

 viśoddhya

4

†uttarārdhena bhājitaṁ† | uttaram

4

 anena bhājitaṁ

4
2

 jātaṁ

2

†labdham sarūpa† | esha rūpādhikam

3

 eśa kāla

ā°	3	u°	4	pa°	3	rūpoṇa	karaṇena	phalam rū°	21
	1		1		1				

dvitīyasya trai-rāśikena

1
1

 di°

7
1

3
1

 di°

pha°
rū°

 21

B 3. [7 verso.] Folio 7 is a very interesting sheet. The writing may be classed as α4. On examining the original I noted that it was a double sheet, but the reproduction (Plate vi) might lead one to conclude that it was a palimpsest. Probably, however, the writing underneath is showing through, or the faint writing marks have been impressed from the contiguous leaf. The two sides are definitely disconnected by their contents and the right side has now been definitely located between folios 6 and 65. Folios 7 (verso), 8 and 9 are difficult to place. Indeed there seems to be some duplication. Folio 5 certainly follows folio 4 and section C cannot very well include folios 7 (verso), 8 and 9.

i. The problem is $7t = ((t-1)\frac{1}{2} + 3)t$ whence $t = \frac{2(7-3)}{4} + 1 = 3$. By the *rūpoṇa* method $s = [(3-1)\frac{1}{2} + 3]t = 21$ and by the 'rule-of-three' $1 : 7 :: 3 : 21$.

The phrases *ādim viśoddhya*, *uttarārdhena bhājitaṁ* and *labdham sa rūpa* are quotations from a lost *sūtra*. Compare with fol. 8 recto.]

B 3—contd.

esha ssamadhanā jātā ||

ii. udā° || ādyeka uttara dvayam dvitīya pañcha pratyaham |
kena kālena samatām vada me gaṇakottama ||

ā°	1	u°	2	pa°	0	niyata nityam	5
	1		1		1		1

†ādīm viśoddhyā†

B 3. ii. The problem is $5t = ((t-1)\frac{5}{2} + 1)t$ whence $t = <2(\frac{5-1}{2}) + 1 = 5$ and $s = 25 >$.

B 4.

i. yojana pañchakam | sapta dināni ^{4 recto.}

tasyaiva gatasya | parata dvitīya nava yojanaika gatake . . . tam

1	di°	5	yo°	dina	7	gatasya	gata	yojana	35	dvi°	1	di°	9	yo°
1		1			1				1		1		1	

†gatisyaiva viśesham cha† . . . yate | gati 5 | 9 | viśesham | 4 |

vibhaktam

1
4

 pūrva gata 35 esha pāder guṇitam

35
4

 . . . bhir

dinai sama gatī bhavanti nava yojanam ||

pratyaya trai-rāśikena	1	di°	5	yo°	35	. . .
	1		1		4	
	1	di°	9	yo°	35	
	1		1		4	
					pha°	. . .

ii. udā° || aṣṭā-daśa yojanā ekena dine yāti |

tasyāṣṭa dinā gatasya |

dvitīya pañcha-vimśe yojanā dine yāti |

B 4. [4 recto.] The writing changes, due possibly to the use of a different pen, but it is different and may be termed α3. This leaf is closely connected with fol. 3 recto and with fol. 5.

i. The example may be restored : One goes at the rate of 5 yojanas for 7 days and then a second starts at the rate of 9 yojanas a day. When will they have traversed equal distances ?

The phrase *gatisyaiva viśesham cha* is a quotation from *sūtra 15* (fol. 3 recto) and *pūrva gata* is a reference to the same rule.

The solution is $t = \frac{7 \cdot 5}{9 - 5} = \frac{35}{4}$ days. 'Proof by the rule of three' $1 : 5 :: \frac{35}{4} : \frac{175}{4}$ and $1 : 9 :: \frac{35}{4} : \frac{315}{4}$ <and $\frac{315}{4} - \frac{175}{4} = 35>$.

One travels at the rate of 18 yojanas in one day for a period of 8 days. A second goes at the rate of 25 yojanas in one day. Determine in what time.

The eleventh leaf must have been close by : indeed *pūrvapi* seems to indicate that it was 'just before.'

B 4—contd.

kena kālena sāsyatām ||

evam ekā-daśama pattre bhilikhita pūrvepi ||

pañcha-daśama sūtram 15

iii. sūtram || ādyor viśesha kartavyam uttarasya viśeshataḥ
vibhaktam muttare

i. 4 verso.

uttaram

2
1

 vibhaktam

1
2

 ādi śesha

2
1

 jātā

1
1

 dviguṇam

2

 rūpa samyutam

3

 esha samkalite

pratyaya | padam . inā ubhaye sthāpitavyā rūponā karaṇe phalam 21 . .
21 dvi

kim prabhūtepi likhite || shodaśama sūtram 17 sūtre bhrāntim asti

ii. sūtram || ādyor viśesha dviguṇam chaya suddhir vibhājitaḥ
rūpādhiḥ kam tathā kālam gati sāsyam tadā bhavet ||

iii. udā° || dvayādi tṛi chayaś chaiva dvitīya tryādi-k-ottaraḥ
dvayo cha bhavate pañthā kena kālena sāsyatām ||

sthāpanam kṛiyate ||

\bar{a}° 2	u° 3	pa° 0		dvi°	\bar{a}° 3	u° 2	pa° 0
1	1	1			1	1	1

karaṇam | ‡ ādyor viśesha ‡

B 4. iii. The rule means (?) $t=2 \frac{(a_1-a_2)}{d_1-d_2} + 1$. Note that the next *sūtra*, on the reverse, commences with the same phrase *ādyor viśesha*.

[4 verso.] i. The example was $a_1=4$, $d_1=3$; $a_2=6$, $d_2=1$. Where a_1 and a_2 are the first terms and d_1 and d_2 are the increments of arithmetical progressions, the sums of which were equal. Therefore $(t-1)\frac{d_1}{2}+4=(t-1)\frac{d_2}{2}+6$ whence $t=2 \frac{(6-4)}{3-1}+1=3$.

The proof is by the *rūpona* method, namely, $s_1=((3-1)\frac{3}{2}+4)3=21$ and $s_2=((3-1)\frac{1}{2}+6)3=21$. But 'why should it be written out in full?' See Part I, § 73.

The remark that the *sūtra* is wrongly numbered was probably added later by some one other than the original scribe. The next *sūtra* is numbered 18 (fol. 5) and so on. This is not a copyist's error: it is one of an original MS.

ii. The rule is much the same as the previous one and means that $t=2 \frac{(a_1-a_2)}{d_1-d_2} + 1$ when $((t-1)\frac{d_1}{2}+a_1)t=((t-1)\frac{d_2}{2}+a_2)t$. The rule is quoted below and on fol. 5 recto.

iii. The example gives $a_1=2$, $d_1=3$; $a_2=3$, $d_2=2$ < whence $t=3$ and $s=15$ >.

C 1.

\bar{a}° 5 1	u° 6 1	pa° 0 1	dha° 0 1
\bar{a}° 10 1	u° 3 1	pa° 0 1	dha° 0 1

5 recto.

karaṇam | †ādyor viśesham† | ādi †chaya śuddhi† chayaṁ
 6 | 3 | śuddhi $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ ādi śesha 5 dviguṇam $\begin{bmatrix} 10 \end{bmatrix}$ uttara viśesha 3
 vibhaktam $\begin{bmatrix} 10 \\ 3 \end{bmatrix}$ sa rūpaṁ $\begin{bmatrix} 13 \\ 3 \end{bmatrix}$ anena ka . . . samadhanā bhavanti |
 pratyayaṁ | rūpoṇā karaneṇa phalaṁ $\begin{bmatrix} 65 \\ dvi^\circ 65 \end{bmatrix}$ esha padam
 aṣṭādaśama sūtram 18

ii. (sūtram) || †dina gamanam ādi rahitam dviguṇam tachchottarena samyutam |
 pratinihita ātmaguṇam jñeyam kshepa samjñako rāśi |
 aṣṭottara guṇite kshepa samjñako datvā mūlaṁ
 pratinihita yutam dviguṇottara bhājitam
 hataṁ 30 . . . †dina gama- 5 verso.
 nam ādirahitam† dina gamana yojanaḥ pañcha $\begin{bmatrix} 5 \end{bmatrix}$ ādi $\begin{bmatrix} 3 \end{bmatrix}$ rahitam
 jātam $\begin{bmatrix} 2 \end{bmatrix}$ †dviguṇam† $\begin{bmatrix} 4 \end{bmatrix}$ †tachchottareṇa samyutam† $\begin{bmatrix} 8 \end{bmatrix}$. . .
 †ātmaguṇam† $\begin{bmatrix} 64 \end{bmatrix}$ eśa †kshepa samjñako rāśi† aṣṭottara saṁgu . . .
 labdha rāśi $\begin{bmatrix} 30 \end{bmatrix}$ aṣṭa guṇam $\begin{bmatrix} 240 \end{bmatrix}$ uttareṇa guṇam uttaram $\begin{bmatrix} 4 \end{bmatrix}$
 guṇitam jātam $\begin{bmatrix} 960 \end{bmatrix}$ †kshepa samjñako datvā† | tatra kshepa samjñ . . .
 64 | yutam jātam $\begin{bmatrix} 1024 \end{bmatrix}$ asya mūlaṁ $\begin{bmatrix} 32 \end{bmatrix}$ †pratinihita†
 8 | yutam jātam $\begin{bmatrix} 40 \end{bmatrix}$ u

C 1.

[5 recto.] The writing is the same as that on folio 4, namely α3, but it changes again in the middle of 5 recto.

i. The example is $a_1=5$, $d_1=6$; $a_2=10$, $d_2=3$, where $((t-1)\frac{a}{2}+5)t=((t-1)\frac{d}{2}+10)t$; and the solution is $t=2(a_2-a_1)/(d_1-d_2)+1$ or $2(10-5)/(6-3)+1=\frac{17}{3}$.Proof by the rūpoṇa method $s_1=<((\frac{13}{3}-1)\frac{a}{2}+5)\frac{1}{3}=((\frac{13}{3}-1)\frac{d}{2}+10)\frac{1}{3}>=65$.

The sūtra number should probably be 17. See fol. 4 verso.

ii. The writing now changes to what may be termed the α4 style. The rule means that < if $DT+Dt=((t-1)\frac{d}{2}+a)t$ > then

$$t = \frac{\sqrt{(d-2(a-D))^2 + 8dDT} + d - 2(a-D)}{2d}$$

where D and T are fixed quantities and a , d and t are elements of an arithmetical progression of which a and d only are given.The quantity designated pratinihita 'set aside' is $d-2(a-D)$, while the kshepa samjñako rāśi 'the quantity known as kshepa' is $\{d-2(a-D)\}^2$ [5 verso.] Writing α4. Notice a semi-looped medial σ near the end.i. The example is < $D=5$, $T=6$, $a=3$, $d=4$; hence $t=\frac{\sqrt{(2(5-3)+4)^2 + 8.4.5.6} + 2(5-3) + 4}{2.4}$ >. The solution proceeds step by step thus: $DT=5.6=30$, $D-a=5-3=2$, $2(D-a)=4$, $2(D-a)+d=4+4=8$; $(2(D-a)+d)^2=64$ and 'this is known as the kshepa quantity'; $SDT=240$, $SDTd=960$; $SDTd+(2(D-a)+d)^2=1024$; $\sqrt{1024}=32$; $2(D-a)+d+32=40$; and < $\frac{40}{8}=5$ >.

Almost the whole of the sūtra on 5 recto is quoted here and on the following pages.

C 2.

i. śike pratyayam |

1	5	5
1	1	1

 phalam anenas saha 55 eśa 6 recto.
samābdhānam ||

ii. udā° || ādi pañcham uttaram *trīṇi* naro yojana gamyate |
dvitiya pratidinams sapta gatasya dina pañchakam |
kena kālena samatām katthyatam gaṇakottama ||

ā°	5	u°	3	pa°	0	prati° gati	7	dina	5
	1		1		1		1		1

pañcha dina ga yojanikam yojana | 35 |

karanam | †dina gamanam ādi rahitam† tatra dina gamanam | 7 |

†ādi rahitam† ādi 5 rahitam

i. . . . anena guṇitam jātam | 840 | †samjñako datvā† tatra kshepa rāshi | 49 | 6 verso.
datvā jātam | 889 | dāna dadāti samam | karanī kriyate

ii. sūtram || akṛite *slishṭha kṛityūnā śesha chchhedo* dvi-saṃguṇaḥ
tad vargaḥ dala saṃslisṭha hṛiti *śuddhi kṛiti kshayaḥ*
anena sutreṇa *slisṭha mūlam ānaya svamatimā*

ii. . . labdham mūlam |

29
48
58

 †pratinihitam† | 7 | anena yutam |

36
48
58

. . . |

2136
58

 †dviguṇottara bhājitam† | tato

C 2. [6 recto.] i. Continues the example. 'Proof by the rule of three' 1 : D :: t : Dt or 1 : 5 :: 5 : 25 and DT+Dt=30+25=55.

ii. The next example is D=7, T=5, a=5, d=3; hence $t = \frac{\sqrt{(2)7-5)+3)^2+8.3.7.5+2(7-5)+3}}{2.3}$

Part of the solution is lost < DT=35, 2(D-a)+d=7, 7²=49 >. It is continued on 6 verso.

[6 verso.] i. Continues the solution: 8DTd=840; 8DTd+(2(D-a)d)²=889. Here the solution breaks off in order to tackle the problem of obtaining the root of a surd quantity, and a subsidiary (un-numbered) *sūtra* is given.

ii. The rule recurs on folios 56 recto and on 57 verso, and with the help of these other versions it has been restored as above.

The rule means that an approximate root of $\sqrt{A^2+b}$ is $A + \frac{b}{2A}$ and that the difference between the squares of these two quantities is $(\frac{b}{2A})^2$; and that by continuing the process closer approximations can be obtained. For a discussion of this rule see Part I, §§ 68, 69, 85. The three versions as they now stand are—

akṛite śli chchhedo dvi-saṃguṇaḥ 6 Verso.

tad vargaḥ dala | saṃslisṭha | hṛit yaḥ

. kṛityūnā śesha chchhedo dvi-saṃguṇaḥ | 56 Recto.

tad varga ślisṭhaḥ hṛiti śuddhi kṛiti kshayaḥ ||

akṛite ślisṭha kṛityūnā śesha chchhedo dvi 57 Verso.

. varga dala saṃslisṭha hṛiti śuddhi kṛiti kshayaḥ

iii. The solution is resumed: < since 889=841+48=29²+48 > the first approximation to $\sqrt{889}$ is 29 $\frac{4}{5}$ and (terming this q₁) we have q₁+2(D-a)+d=29 $\frac{4}{5}$ +7=36 $\frac{4}{5}$ = $\frac{1832}{5}$ < and t₁= $\frac{2136}{59 \times 6}$ = $\frac{178}{23}$ > where t₁ is an approximate value depending on q₁.

C 3.

6	447	dalitā	447	sāsyē yutam	737	pada 7 recto.
1	29		58		58	
8						
60*		ghnā	tatra padam	178	anena guṇitam jātam	65593
16 cha°				29		841
60*						
33 li°		śli . . .	tya śesham kṛiyate	65569	bhāge hṛite	
60*				841		
6 vi°						
60*		pratyayam	trai-rāśikena	1	7 yo°	178
śe° 6				1	1	29
29						phalam
yojana 42	śe	28	niyatam tena . . .	77		
		29				

ekona-vimśatima sūtram 19 ||

C 3. [7 recto.] This continues the example started on fol. 6 recto. [The numbers marked with asterisks are change-ratios (see Part I, §§ 103—105).] The set of figures on the left expresses $\frac{178}{29}$ as a sexagesimal fraction (see Part I, § 58), i.e., $\frac{178}{29} = 6 + 8^1 + 16^{11} + 33^{111} + 6 \frac{29}{29}^{17}$. The portion of the statement above the 16 is missing but the restoration is certain. Of the abbreviations *cha°* has not yet been identified; *li°* stands for *liptā* (Gk. λεπτα); *vi°* for *viliptā*; *śe°* for *śesham* 'remainder.' In Hindu astronomical works *liptā* means a 'minute of arc,' and *viliptā* 'a second of arc.' This use of the sexagesimal notation for arithmetical purposes in an Indian work is unique. The solution proceeds to find the approximate value of s_1 which depends on t_1 and ultimately q_1 . We have $s_1 = ((t_1 - 1) \frac{d}{2} + a)t_1$. Now $(t_1 - 1)d = (\frac{178}{29} - 1)3 = \frac{447}{29}$; $(t_1 - 1) \frac{d}{2} = \frac{447}{58}$; $(t_1 - 1) \frac{d}{2} + a = \frac{447}{58} + 5 = \frac{737}{58}$; and $((t_1 - 1) \frac{d}{2} + a)t_1 = \frac{737}{58}$. $\frac{178}{29} = \frac{65593}{841}$.

But $DT + Dt_1 = 7(5 + \frac{178}{29}) = \frac{65593}{841} = 77 \frac{512}{841}$.

'Proof by the rule of three': $1 : 7yo° :: \frac{178}{29} : 42 \frac{28}{29}$ and $< 48 \frac{28}{29} + 35 = 77 \frac{512}{841} >$.

[Note that $\frac{65593}{841} - \frac{65593}{841} = \frac{24}{29} = (\frac{48}{29})^2 \times \frac{1}{8.3} = \frac{61}{60}$. This process of reconciliation is explained in Part I, § 85.]

The *sūtra* number should probably be 18. See fol. 4 verso.

C 4.

ā°	1	u°	1	pa°	0	60
	1		1		1	1

65 verso.

karaṇam		†ashtottaraghnē guṇite†		ashta ghanam		480		uttara
ghana . . .		dvi-ghnam ādi		ādi dvi-guṇa		2		chayojjhitam
.		uttaram		ato uttaram pātayitvā ekaṁ bhavati		1		va . . .
nikshipya dhanasya		481		mūlam ślishṭha karaṇyā		21		
						40		
						42		
vaṁśam		882		śesham chatvāriṁśa prithak sthāpya		40		
		40						
		42						
yojyam		922		tan mūla varjitam	tan mūlam	880		
		42						

C 4. [65 verso.] Folio 65 consists of two leaves stuck together. The writing on both sides may be classed as α4. The left side has no direct connexion with fol. 7 recto but it belongs to the same section.

The *sūtra* here quoted from is lost, or hidden, for possibly when folios 7 and 65 are separated it may be discovered. It may be said to be one of the most important *sūtras* of the whole work judging by the care and elaboration with which it is illustrated. It must mean that < when $s = ((t - 1) \frac{d}{2} + a)t$ then $t = \frac{\sqrt{(2a-d)^2 + 8d} - (2a-d)}{2d}$ > where a , d , t and s are respectively the first term, the common difference, the number of terms and the sum of an arithmetical progression.

The example is $a=1$, $d=1$, $s=60$; hence $t = \frac{\sqrt{(2-1)^2 + 8.1.60} - (2-1)}{2.1} = \frac{\sqrt{481}-1}{2}$.

The solution proceeds $8ds=480$, $2a-d=1$, $(2a-d)^2+8ds=481$; by the square-root method (see fol. 6 verso) the first approximation is $21 \frac{40}{42} = \frac{882+40}{42} = \frac{922}{42}$ and $< t_1 = (\frac{922}{42} - 1) \div 2 = \frac{880}{54} >$.

C 5.

880 84	964 168	guṇita jātam	848320 14112	chatvāriṃśa prithak sthānām vargam	56 verso.
-----------	------------	--------------	-----------------	------------------------------------	-----------

1600	esha uparā pātya śeṣam	846720 14112	vartya jātam	60
------	------------------------	-----------------	--------------	----

21	teshām varggah tasthāt	56 recto.
20		
21		

akṛite śliṣṭha kṛityūnān śeṣa chchhedo dvi-saṃguṇam |

tad varga dala saṃśliṣṭhaḥ hṛiti śuddhi kṛiti kṣayaḥ ||

†śeṣa chchhedo dvi-saṃguṇa† kṛi

21	21 bha
20	400 dala 1 saṃśliṣṭhaḥ 20
21	441 2 21 +

śeṣam pātya dvā bhājita . . . †adham upare uparam†
guṇitavyam vargam yāva marjaye

425042	400	śeṣam	424642
19362	19362		19362

O 6. [56 verso.] Continues the example. $s_1 = ((t_1 - 1)^{\frac{1}{2}} + 1) t_1 = t_1 \frac{(t_1 + 1)}{2} = \frac{840}{91} \cdot \frac{964}{168} = \frac{816720}{14112}$, but $< \frac{c_1}{84} = \left(\frac{40}{2.21} \right)^2 / 8. > = \frac{1600}{14112}$ and $\frac{848320 - 1600}{14112} = \frac{816720}{14112} = 60$.

The bottom half of fol. 56 verso is blank but the example is continued on 56 recto.

[56 recto.] This continues the example given on fol. 65 verso. The top part of the leaf is much broken up; but the square-root rule (see fol. 6 verso) is given. Why this rule is repeated is not quite understood nor is it understood why it comes between two approximations of the same surd. Anyhow the general aim is clear: since the first approximation is $21\frac{2}{3}$ the second is given by

$$q_2 = 21\frac{2}{3} - \frac{1}{3} \cdot \left(\frac{2}{3} \right)^2 / 21\frac{2}{3} = 21\frac{2}{3} - \frac{1}{441} \times \frac{21}{401} = \frac{421,612}{19362}$$

C 6.

405280	444004	ardham kartavyam	64 recto.
38724	38724				

405280	444004	saṃguṇya jātam	a	hrarā hareshu guṇ
38724	77448			

179945941120	asya ūrdham	160000 +
2999096352			

O 6. [64 recto.] and $t_2 = \left(\frac{421612}{19362} - 1 \right) \div 2 = \frac{405280}{38724}$. Also $s_2 = \frac{t_2 (t_2 + 1)}{2} = \frac{405280}{38724} \cdot \frac{444004}{77448} < \frac{179,945,941,120}{2,999,096,352} >$ and $s_2 - \frac{160,000}{2,999,096,352} = \frac{179,945,781,120}{2,999,096,352} = 60$.

C 8.

10225	dalitā	10225	ādi yutaḥ	108625	padaghñā 45 recto.
32800	.	65600			65600	

pada samyutā 6455040625
3227520000 ato pañcha-viñśa uparāḥ

6455040000 labdham 2 esha dhanam ||
3227520000

ā°	1	u°	1	padu	0	dhanu	7000
	1		1		1		1
	2		2				

. 384 asya varga 147456 akṛi 21743271936 45 verso.

esha sarva guṇitā karaṇi kṛitvā bhājita jātāḥ 1158 + amśair
671250

amśā guṇaye raśi varjya jātāḥ

579		579	
768	294912	515520000	294912 +
1158	777307500	777307500	777307500

śesham	579	450576267588
	515225088		777307500
	777307500		

dvayena mūle

C 8. [45 recto.] i. The greater portion of this example is lost, but can be restored. The example was $a=1\frac{1}{2}$, $d=1\frac{1}{2}$, $s=2$; whence $t = \frac{\sqrt{10}-3}{6}$. The first approximation to $\sqrt{10}$ is $q_1=10\frac{1}{4}$ and the second is $q_2=10\frac{1}{4}-\frac{1}{2}(\frac{1}{4})^2/10\frac{1}{4}=10\frac{81}{328}$. This gives $t_2 = \frac{10\frac{81}{328}-3}{6} = \frac{59425}{49200}$, and $s_2 = ((\frac{59425}{49200}-1)\frac{1}{2} + \frac{1}{2}) \frac{59425}{49200} = (\frac{10225}{32800} \cdot \frac{1}{2} + \frac{1}{2}) \frac{59425}{49200} = (\frac{10225}{65600} + \frac{1}{2}) \frac{59425}{49200} = \frac{108625}{65600} \cdot \frac{59425}{49200} = \frac{6,455,040,625}{3,227,520,000}$. < Now $\frac{e_2}{s_2} = \frac{625}{3,227,520,000}$ (see Part I § 86 (v)) > and $s=s_2-\frac{e_1}{s_1} = \frac{6,455,040,625-625}{3,227,520,000} = 2$.

ii. The statement without any formal question should be noted. The example is $a=1\frac{1}{2}$, $d=1\frac{1}{2}$, $s=7000$. The first part of the solution is lost but a good deal of the later working is preserved on folios 45 verso and 46 recto. We have $q_1=579\frac{768}{1168}$. (See part I, § 86 (vi).)

45 verso. The second approximation is given by $q_2=579\frac{768}{1168}-\frac{1}{2}(\frac{384}{579})^2/579\frac{768}{1168}=579\frac{768}{1168}-\frac{(384)^2}{579} \cdot \frac{1168}{671250}=579\frac{768}{1168}-\frac{294,912}{777,307,500}=579\frac{515,520,000-294,912}{777,307,500}=579\frac{515,225,088}{777,307,500}=\frac{450,576,267,583}{777,307,500}$. Continued on folio 46 recto.

C 9.

448244345088 4663845000

443580500088 4663845000

221790250044 1554615000

dalitā e . 46 recto.

110895125022 1554615000

ādi samyuta

113227047522 1554615000

pada-ghnā

50753383762746743271936 7250483394675000000
--

. Karāṇi pāta

21743271936

pātita jātā uparānyāsa sthāpa . . .

507533837627250000000000 7250483394675000000

bhā 7000

C 9. [46 recto.] Continued from 45 verso. $t_2 = \left(\frac{450,573,225,555}{777,307,600} - 3 \right) \div 6 = \frac{448,244,345,088}{4,663,845,000}$ and $t_2 - 1 = \frac{448,244,345,088}{4,663,845,000}$, $(t_2 - 1) d = \frac{221,200,250,044}{1,551,615,000}$, $(t_2 - 1) \frac{d}{2} = \frac{110,833,125,022}{1,551,615,000}$, $(t_2 - 1) \frac{d}{2} + a = \frac{118,252,047,522}{1,551,615,000}$ and finally $s_2 = ((t_2 - 1) \frac{3}{2} + \frac{1}{2}) t_2 = \frac{50,753,353,762,725,000,000,000}{7,250,483,394,675,000,000}$. New $\frac{c_1}{s_1} = \frac{21,743,271,936}{7,250,483,394,675,000,000}$ and $s = s_2 - \frac{c_2}{s_1} = \frac{50,753,353,762,725,000,000,000}{7,250,483,394,675,000,000} = 7000$.

D 1.

<i>maḍe</i> 8	<i>maḍe</i> 6	<i>maḍe</i> 3
---------------	---------------	---------------

. *kā* 20 *apara prashtaḥ pārā* 46 verso

a i e vihujaṇa vī ha . . hai . . . ṇa | gore jā ma cha | uppaṇe
sā male a . . ḍha pa . ḍhale āpot diṇe āgaṇe vihujaṇa ehu vī
karaṇam | trai-gore varehahipaṇehi sā

D 1. [46 verso.] Writing α 4. Find order 9. This is quite unintelligible to me.

D 2.

tola 5 70^a recto
. $\frac{35}{2}$ | *ete bhāgā* 70^b recto
. $\frac{17}{5}$ | $\frac{117}{70}$
. 2 . 0 *rītā* 7 *pala 2 tola 1 . . . pala 6 ||* 70^c recto
udā^o || samā napeśi kṛitāni cha
dvecha tisraś
tisra samādāya] tulitāni trayo-daśe |
. *ekaikasya sārdhayah . . .*

1	1	1	d
2	3	4	

prakshepa yuktyā phalam 70^a verso
. *ri ri* 70^b verso
gunya phala rāśi
. *katram pala 8* 70^c verso

D 2. * [70]. Folio 70 consists of 5 scraps not obviously connected. The writing may be classed as α₂. The 'find order' is 65 and this and the five following fragmentary leaves are placed in their 'find order,' for want of some more reliable basis of classification.

70 recto is mostly unintelligible but $x(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}) = 13$ and $x=12$ is a solution.

70 verso. Here $x(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}) = 65$ and $x=60$ is obviously connected in some way with the example on 70^c recto but they are two separate examples.

D 2—contd.

udā° || ardha tri . . . dāñśā pañcha śashti nṛipo dadau |

sevakānām tu dī

1	1	1	drishya	65	sadri
2	3	4		1	

.

D 3.

2	2	2	drishya	ato sadriśa . . .	hakaṁ	69 recto.
5	6	7				

upari māmsam taṁḍulā bhavanti chatvālinśa | dūnā chau

rāśi . . . eta taṁḍulā | dvā-chatvāriṁ vanti ete vṛīhakā

sarvattraḥ sthāpanam asya | . . . : .

pratyaya trai-rāśikena	5 ā°	2 tam°	210	pha° tam°	84
	1	1	1		

. . . iyasya kriyate	6
	1	

. yate rāśih	7	2	210	phalam
	1	1	1	

. katram 105 . .

69 verso.

udā° || tribhir dattai triḡuṇā triḡuṇena tu |

. tad uchyatām ||

1	3	9	drishya	130	prakshepa	10	30	90	ekatram	130
1	1	1		1						

. . . vān || tam śatam tribhir datyai paravaptrā pavaptri kai

4	6	9	dri°	190	40	60	90	ekatram	
1	1	1								

D 3. Folio 69 consists of four pieces but is not quite so shabby as folio 70, for the two larger pieces fit together.

[69 recto]. The statement means $x(\frac{3}{2} + \frac{3}{3} + \frac{3}{9}) = 214$ whence $x = 210$. The 'proof by the rule of three' is

$$5 \text{ ā}^\circ : 2 \text{ tam}^\circ :: 210 : 84 \text{ tam}^\circ < \text{and } 84 + 70 + 60 = 214 >$$

$$6 \text{ ā}^\circ : 2 \text{ tam}^\circ :: 210 : 70 \text{ tam}^\circ$$

$$7 \text{ ā}^\circ : 2 \text{ tam}^\circ :: 210 : 60 \text{ tam}^\circ$$

[69 verso]. Here $x(1 + 3 + 9) = 130$ whence $x = 10$ and the numbers are $10 + 30 + 90 = 130$. Again $x(4 + 6 + 9) = 190$ and $40 + 60 + 90 = 190$.

D 4.

168	deśa dvātya	pātya jātā	68 recto,
4			
śeṣam* 21	ekatram 29	dram 2	
bāri			

D 4. [68 recto.] Consists of small fragments which probably belong to folio 67. Writing α_2 . The phrase *pātya śeṣam* occurs on some six other occasions (on folios 31, 62, 63, 56).

D 5.

1	traī-rāśikena	2 dine 1	3 dram° 1 2	168 di 11	31 recto.
tiyasya kṛiyate	3 di° 1	2 dram° 1 2	168 dinā 11	phalam dram°	
140 11	prathamena dattam	saptaḥ dattaḥ	samadhanā jātā		
sadṛiṣam	77 11	294 11	pātya śeṣam†	217 11	dvitīyasya
datta	77 eśas	sama-dhanā jātā			
ii. punānyam sarva	bhā 4 dine	dram° 15 jīvyā			
	1+	4			
dvitīyasya	bhā 3 dine	dram°			
	1				
	3				

D 5. Folio 31 consists of two leaves stuck together and the writing on the two sides differs. The leaf is very ragged.
[31 recto.] The writing may be classed as α_2

i. The example may be restored with some uncertainty: A earns $3\frac{1}{2}$ *drammas* in 2 days, B^r earns $2\frac{1}{2}$ in 3 days, A gives B 7 *drammas* and this makes their possessions equal. How long had they been earning?

< Since $\frac{3\frac{1}{2}}{2}t - 7 = \frac{2\frac{1}{2}}{3}t + 7$ we have $t = \frac{14}{7/4 - 5/6} = \frac{168}{11} = 15\frac{3}{11}$ days. >

Proof by the rule of three 2 days : $3\frac{1}{2}$ *drammas* : : $\frac{168}{11}$ days : $\frac{294}{11}$ *drammas*

and 3 days : $2\frac{1}{2}$ *drammas* : : $\frac{168}{11}$ days : $\frac{140}{11}$ *drammas* !

and $\frac{294}{11} - \frac{140}{11} = \frac{154}{11} = 14\frac{2}{11}$

ii. Another example of the same kind.

D 5—contd.

. . . . *kāraṇam* | chchheda saṁ-guṇe |

dram°	1	4	ya
	1	1	
		2	

 | 31 verso.

dram°	1	6	mudgā
		1	
		2	

 | rdha yutī hṛitī phalaṁ || asya guṇākāro dvayāna

2	1
9	13
2	2

 | †uparam guṇaye† adau tāva dva

2

 | dvi-nava-bhāgesu

sūtram ||

[31 verso.] Some of the lower writing shows through and it is very difficult to differentiate. The word *guṇākāra* ? 'form of multiplication' occurs again on fol. 42 verso.

D 6.

i. chchhesham ta, dviguṇa . tā | 67 verso.

nirgachchha praviśa māne chatvāri dattaḥ

puna dvi-guṇam

sūnya hastam gataṁ tasya kiṁ atra mūlādhana syāt |

1	2	bhā°	1		1	2	bhā°	2		2	2	bhā°	3		3	2	bhā°	4	
1	1		1		1	1		1		1	1		1		1	1		1	

4	2	bhā°	5		1	
1	1		1		4		

D 6. 67. The surface of the leaf is much worn and the writing is in some places rubbed off. The writing is x2.

[67 recto.] i. The example seems to relate to a game at which a certain quantity was staked and eventually all lost. The statement means $1 + \frac{1}{2} (2 + \frac{1}{2} (3 + \frac{1}{2} (4 + \frac{1}{2} (5 + \frac{1}{2} . \frac{1}{2})))) = < \frac{1}{2} >$

D 6—contd.

49	12	jātā	61	.. sadṛśam	8	11	puna	67 recto
8	8		8		1	2	8	

16	61	jātā	77	.. sadṛśam ekasya .	16	yutam	77
8	8		8		16		16

jātam

93
16

 esha phalam bhavati |

pratyayah	93	1+	2	2+	2+	2	3+	3+	2	4+	4
	16	1	1	1	1	1	1	1	1	1	1	

ii. huṇḍikā samānayaṇa sūtram ||

dina bhakta viśeṣam cha dvi-guṇam kriyate chaiva

kālam eṣhām vinirdiśet trai-rāśika vidhānena

. . dattam cha pātavyam† sūkshme dattam cha tatsamam ||

udāharaṇam || dvi-guṇa

[67 verso.] Worked out by steps $< \frac{3}{2} (5 + \frac{1}{2} \cdot \frac{1}{2}) = \frac{33}{4}$, $\frac{33}{4} + 4 = \frac{49}{4} > \frac{1}{2} (\frac{49}{4} + 3) = \frac{61}{4}$, $\frac{1}{2} (\frac{61}{4} + 2) = \frac{77}{8}$, and $\frac{77}{8} + 1 = \frac{93}{8}$ which is the answer.

Proof. $((((\frac{93}{8} - 1) 2 - 2) 2 - 3) - 4) \frac{3}{8} - (5 + \frac{1}{2} \cdot \frac{1}{2}) = 0$.

ii. This huṇḍikā sūtra should be intelligible but it is not yet clear to me.

D 7.

. dvi-guṇam dvi-guṇam bhāram labdham 28 recto

14 || puna kriya

. vet || guṇaye

1
16

1
8

 guṇi jātā 28 recto

ahutva adho guṇa bhāgasya divardhā x kim

1
96

1
1
2

 phalam phalam } 5

D 7. [28]. Find order 60. Writing? α4. Not intelligible.

E 1.

... ekārgham tu panyānām eka-dvi-tri-chatush-shaṭ
 ... panyān imānayaḥ
 sthāpanam kriyate

66 recto.

1 1	dram° 1 2	dram° 1 3	dram° 1 4	dram° 1 6
1 1	1 1	1 1	1 1	1 1

...
 pratyaya trai-rāsikena

66 verso.

1 dram° 1	1 rū° 1	12 dram° 1	phalaṁ rūpa 12
1 dram° 1	2 rūpa 1	6 dram° 1	phalaṁ rūpa 12
1 dram° 1	3 rūpa 1	4 dram° 1	phalaṁ rūpa 12
1 dram° 1	4 rūpa 1	3 dram° 1	phalaṁ rūpa 12
1 dram° 1	6 rūpa 1	2 dram° 1	phalaṁ rūpa 12

E 1. Folio 66 consists of a bad piece of birch-bark containing a large knot. The knot is repeated on folio 53. The find order is 58. Writing is probably α4.

The problem may have been something like this: The rates of purchase are one, two, three, four and six articles for one dramma. What will be the cost of twelve of each?

The cost of one of each would be $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{13}{6}$ therefore the cost of 12 of each is 27 drammās, and the numbers of articles are 12, 6, 4, 3 and 2.

'Proof by the rule of three'

1 dram°	: 1 rū°	: 12 dram°	: 12 rū.
1 "	: 2 "	: 6 "	: 12 "
1 "	: 3 "	: 4 "	: 12 "
1 "	: 4 "	: 3 "	: 12 "
1 "	: 6 "	: 2 "	: 12 "

E 2.

1 yo°	1 di°	6
1	1	1
2	1	

53 recto.

viśeṣaṁ tu tatra gatim 3 2 viśeṣaṁ 1
 2 1 2 . . . sarva gati

E 2. Folio 53 resembles fol. 66 in appearance and has the same large knot. Its find order is not known. Writing ? α4. The problems are, however, similar to that on fol. 9 recto.

[53 recto.] The following conjectural restoration of the problem is offered:

One goes $1\frac{1}{2}$ yojana in a day and another 6 in 3 days. If the first had a start of 9 yojanas when would the second overtake him?

Since $\frac{3}{2}t + 9 = 6t$ we have $t = \frac{9}{\frac{3}{2}} = 18$ days.

'Proof by the rule of three': 1 day : $\frac{3}{2}$ yo° : 18 days : 27 yo° and 27 + 9 = 36

1 " : 2 yo° : 18 " : 36 yo°

E 2—contd.

yojana | 9 | anena guṇaye | 18 | anena bhavishyati

pratyaya trai-rāśikena | 1 di° | | yo° 27 dina 6 ādau yojana 9 .
1 1

| . | 1 | 1 di° | . . yo° phalaṁ yojana
| . | 1 | 1

. . . .	18 yojana	20 dina	phalaṁ yo° 27 53 verso 53 verso
	1	20 ghaṭike	7
		35* gha° dina	
. . . .	27 yo	20	pha° yo 36
		20 ghaṭike	7
		35* ghaṭike dina	

[53 verso.] The following is merely a guess at the problem: One goes 18 yojanas in 96 days and another 27 yojanas in 108 days. The first starts from A and the second from B and the distance AB is 9 yojanas. When will they meet if they go only for $\frac{1}{2}$ or 35 ghaṭikas of each day? (60 ghaṭikas=24 hours).

In one day they together go $\frac{18}{96} + \frac{27}{108} = \frac{7}{12}$, that is they meet at the rate of 1 yojana in $\frac{12}{7}$ days and actually meet each other in $\frac{9 \times 16}{7}$ days = 20 $\frac{4}{7}$ or 20 days 20 ghaṭikas.

Proof 96 days : 18 yo° :: 20d. 20 gha° : $\frac{27}{7}$ yo° and $\frac{27}{7} + \frac{12}{7} = 9$.

108 days : 27 yo° :: 20d. 20 gha° : $\frac{36}{7}$ yo.

E 3.

udā° || śaḍ-vimśas cha tṛi-pañchāśa ekona-tṛimśe vacha |

58 recto

dvā-śa . . . śaḍ-vimśa chatuś-chatvālimśa saptati |

chatush-śashṭi nava mśa naṁtaraṁ |

trir-āśīti ekavimśa aṣṭa pakam |

. . . 296226447064994

E 3. [Folio 58.] Find order not known. Writing ? α4. Possibly two leaves stuck together.

[58 recto.] This gives pairs of numbers, first in words and then in figures, thus :

Twenty-six and fifty-three and one less thirty

. twenty-six, forty-four, seventy

sixty-four

eighty-three, twenty-one, eight

and in figures 29 62 26 44 70 64 99 4

E 3—contd.

58 verso

sthāpanam kriyate . . .	1 1	yuvi 1 1	sūḍha 1 1	ḍrishya 20
. . . 3		mam 1 1 2	mamḍa 1 2	mamḍe 20

ta datta jātam mamḍa 2 yu 5 sūḍhe 1

[58 verso.] There is basis for the following restoration—

A man earns 3 in one day, a young woman $1\frac{1}{2}$ in 1 day and $\frac{1}{2}$ in one day. If 20 earn 20 *maṇḍas* in one day, how many of each will there be?

Let x, y, z be the numbers of each class, then $x+y+z = 20$ individuals

$$3x + \frac{3}{2}y + \frac{1}{2}z = 20 \text{ maṇḍas}$$

of which the only solution in positive integers is that given in the text, namely $x=2, y=5, z=13$. This problem known as the 'Hundred Hens' problem in China, and as the *Regula Virginum*, etc., in Europe is noted upon in Part I, §80 (a).

E 4.

tri-bhāga dine tatha | tri rūpa pañchabhi dinai | 21 recto.

eshām da

rū° 1 1	rū° 1 1	rū° 3 1	ḍrishya 100 1
1 di° 3	1 di° 2	5 di 1	

karaṇam 	kṛitvā	3 1	2 1	3 5	dri° 100 1
----------------------	--------	--------	--------	--------	---------------

4. Folio 21 consists of 7 scraps of which the largest piece is partly intelligible. The find order is 55 and the writing α1, 4.

[21 recto.] Apparently this means: 1 rū° is given or obtained in $\frac{1}{3}$ days, 1 in $\frac{1}{2}$ day and 3 in 5 days by three separate individuals (or classes) and the total amount given or obtained is 100.

In one day $\frac{1}{3} + \frac{1}{2} + \frac{3}{5} = 3 + 2 + \frac{3}{5} = 5\frac{3}{5}$ is given, so that one is given in $\frac{5}{8}$ days and 100 in $\frac{500}{8} = 17\frac{5}{8}$ days.

E 4—contd.

. vārdham tritīyasya 21 verso,
 jīva-lokāt eshām dīnār kasya kim bhavati ||

2 dī°	3 dī°	4 dī°
1	1	1
2	2	2
1 dī°	1 dī°	1 dī°
1	1	1
2	3	4

. parivartanam kṛiyate	10 6	21 8	36 10	dī 500 1
--------------------------------	---------	---------	----------	-------------

prakshe

[21 verso.] Here the main elements of a problem are preserved and the problem is continued on folio 22. The problem probably was to the effect that: A gave $2\frac{1}{2}$ dīnāras in $1\frac{1}{2}$ days, B gave $3\frac{1}{2}$ in $1\frac{1}{2}$ days and C $4\frac{1}{2}$ in $1\frac{1}{2}$ days. In what time would they have given 500 dīnāras?

In one day $\frac{2\frac{1}{2}}{1\frac{1}{2}} + \frac{3\frac{1}{2}}{1\frac{1}{2}} + \frac{4\frac{1}{2}}{1\frac{1}{2}} = \frac{10}{6} + \frac{21}{8} + \frac{36}{10} = \frac{947}{120}$ is given. Therefore 500 is given in $\frac{500 \times 120}{947} = \frac{60000}{947} = 63\frac{230}{947}$ days>

Continued on fol. 22 recto.

E 5.

.

473500 947

 vartita jātā phalam dī 500 ||

22 recto

asya pratyaya trai-rāśikena

2 dī°	1 dī°	100000 dī°	phalam di 60000
1	1	947	947
2	2		
3 dī°	1 dī°	157500 dī°	phalam di 60000
1	1	947	947
2	3		
4 dī°	1 dī°	216000 dī°	phalam di 60000
1	1	947	947
2	4		

E 5. [22 recto] continues the solution of the example on fol. 21 verso.

<The gifts are therefore $\frac{100,000}{947} + \frac{157,500}{947} + \frac{216,000}{947} = \frac{473,500}{947} = 500$ dīnāras.

* Proof of this by the rule of three" $2\frac{1}{2}$ dī° : $1\frac{1}{2}$ days :: $\frac{100,000}{947}$ dī : $\frac{60,000}{947}$ days:

$3\frac{1}{2}$ " : $1\frac{1}{2}$ " :: $\frac{157,500}{947}$ " : $\frac{60,000}{947}$ "

$4\frac{1}{2}$ " : $1\frac{1}{2}$ " :: $\frac{216,000}{947}$ " : $\frac{60,000}{947}$ "

F 1.

- i. dvi-guṇaṁ dvitīyasya prathama tīya . . . (prathamā 22 verso.
 chaturguṇaṁ chaiva chaturthe chaiva dattavān cha śatam ekaṁ
 dvayānvayaṁ ||| vadasva prathame dattaṁ kiṁ pramāṇāṁ . . . sya .

0	2	3	4	drishya	200
1	1	1	1		1

†sūnyam eka-yutaṁ kṛtvā† 1 | 2 | 3 | 4 | . . . †kshepa yuktyā†
 phalaṁ || 20 | 40 | 60 | 80 | evaṁ 200 || eshāṁ

ā° 20	u° 20	pa° 4	rūpoṇā karaṇena phalaṁ 200
1	1	1	

- ii. sūtraṁ || yadrichchha pinyase sūnye tadā vargaṁ tu kārayet

F 1. [22 verso.] i. This appears to be the beginning of a new section. The sūtra is lost. Find order 54, writing α4.

The problem was something like this: A certain amount was given to the first, twice that to the second, thrice it to the third, and four times to the fourth. State the amount given to the first and the shares of the others, if the total amount given was 200.

The shares are represented by 0, 2, 3, 4. 'Having added one to the nought' the sum is $1+2+3+4=10$. <Then the proper share of the first is $\frac{200}{10}=20$ > . Having added in this value the series becomes $20+40+60+80=200$.

The proof by the rūpoṇa method gives $<((4-1)\frac{2}{3}+20)4>=200$.

For the method of solution, the *regula falsi*, see Part I, §§71 and 72, and for the rūpoṇa method see §73. The whole section is dealt with in §87, and the use of the symbol for 'nought' in §60.

ii. The sūtra begins "Put what number you please in the empty place (or for the nought)." This is quoted on fol. 23 recto and so is tadā vargaṁ tu kārayet, etc.

F 2.

- i. cha tri-guṇaṁ 23 recto.

. prathamasya tu kiṁ bhavet

0	tadā 2	tadā
1	1	

†yadrichchhā vinyase sūnye† . . chchhā | 1 | †tadā vargaṁ tu kārayet†

1	2	2	3	6	prakshipe guṇitaṁ 1	2	6	24
1	1	1	1	1				

. . prakshiptaṁ 33 ||| drishyaṁ vibhajet | 132 | vartyaṁ jātaṁ | 4 |
 33 1

F 2. [23 recto.] The find order is 52.

i. The example may be represented by $x+2T_1+3T_2+4T_3=132$: Where T_1, T_2 , etc., represent the values of the first, second, etc. terms. Make $x=1$ then the terms are $1+2+6+24=33$ and the proper value of x is $\frac{132}{33}=4$ and the series becomes $4+8+24+96=132$.

All the technical terms here employed are of interest and will be dealt with in due course: *ichchhā* 'an assumed number'; *varga* 'a series'; *prakshepa* 'something thrown in' or 'an interpolation'; *vartya* 'cancelled'; *drishya* 'the given number'; etc.

F 2—contd.

. dattam || ato nyāsaḥ | 4 | 8 | 24 | 96 |

esha varga krama gaṇitam || atha yuti vargam kṛi

ii. sūtram || kāmikaṁ sūnye vinyastam tadā chaiva krame guṇam

i. kṛitvā chaturtha

23 verso

prathamasya tu kiṁ bhavet

0	2	1	3	3	12	4	dri°	300
1	1	1	1	1	1	1		1

†kāmikaṁ sūnye piṇyastam† kāmikaṁ 1 || esha nyastam

†tadā chaiva krameṇa guṇitam† | 1 | 2 | 9 | 48 | eshām yu . | 60 |

anena dṛishyam bhājitam

1	300
60	1

 jātā

5

 e

anena kshepaṁ guṇaye | 5 | 10 | 45 | 240 | yuti

varga gaṇitam ||

ii. udā° || prathamasya na dattam chaivā dhānam |

sa cha dvyārdha yuta dhānam

ii. The term *kāmika* is practically synonymous with *icchā* or *yadricchā* 'what you please'; 'an assumed number.' Bhāskara uses *ishṭa* much in the same way. A good deal of the *sūtra* is quoted on fol. 23 verso.

[23 verso.] i. The example may be represented by $x+2T_1+3(T_1+T_2)+4(T_1+T_2+T_3)=300$. Put $x=1$ then the series becomes $1+2+9+48=60$ and the proper value of x is $\frac{300}{60}=5$ and we have $T_1=5$, $T_2=10$, $T_3=45$, $T_4=240$ and $\Sigma T=300$.

ii. The example is solved on fol. 24 recto.

F 3.

śataṁ chatuś-chatvalimsā **dattam chaiva chaturguṇam**

24 recto

kiṁ prathamasya

0	1	2	2	3	3	4	4	dri°	144
1	1	1	1	1	1	1	1		1
	2		2		2		2		2

F 3. [24 recto.] The example may be represented by

$$[x(1+T_1)]+[2T_1+2\frac{1}{2}x]+[3T_2+3\frac{1}{2}x]+[4T_3+4\frac{1}{2}x]=144\frac{1}{2}$$

Set $x=1$ and the series becomes $\frac{5}{2}+\frac{1}{2}+\frac{5}{2}+\frac{3}{2}=\frac{7}{2}=144\frac{1}{2}$ which is the same as the given sum and therefore $x=1$ is correct.

The phrase marked ** is deleted in the original. The expression "*upare uparam adhe adham guṇaye*" is obviously quoted from a well known rule relating to fractions: 'numerator should be multiplied by numerator and denominator by denominator.' See also C5, D5.

F 3—contd.

. sūnyeśu $\frac{1}{1}$ | yutam chaiva guṇam | tatah
 yutam chaiva guṇam kṛtvā kāraye gaṇa $\frac{5}{2}$ guṇam | upare
 uparam adhe adham guṇaye | $\frac{10}{2}$ | sārḍha dv . . yutam . tiya rāśyā guṇanam |
 sārḍhais saptabhi trīṇi | $\frac{45}{2}$ | sārḍha traya yutam . . chaturtha rāśi
 guṇayesh śaḍviṃśatibhi | jātā | $\frac{208}{2}$ | sārḍha chatvāri yu
 $\frac{289}{2}$ | evaṁ dṛśyam | sarvam tadeva jātam

i. tri-sārḍha yu

24 verso.

. . chatur-guṇam chaturthena navārdha yutam dattam |
 . . dviśatā dvāviṃśādhikā kim atra prathamasya dattāsit

0 3	2 5	3 7	4 9	ekatram dattam 222
1 2	1 2	1 2	1 2	

śūnya datvā | 1 | yuta guṇita yuta krameṇa jātam |||

sthāpā $\frac{5}{2}$ | $\frac{15}{2}$ | $\frac{67}{2}$ | $\frac{357}{2}$ | dṛishya 222 | prakshepena

jātam 222 ||| . . dṛiśyāḥ 222 ||

ii. udā° ||| prathamam na jānāmi | divardha yutam

[24 verso]. i. The example may be represented by

$[x(1+\frac{1}{2})]+[2T_1+\frac{5}{2}x]+[3(T_1+T_2)+\frac{3}{2}x]+[4(T_1+T_2+T_3)+\frac{1}{2}x]=222;$

Set $x=1$ and the series becomes $\frac{5}{2}+\frac{15}{2}+\frac{67}{2}+\frac{357}{2}=222.$

The same quotation *sūnya sthāne* . . . *rūpam datvā* occurs on fol. 25 verso. See also at the bottom of fol. 26 recto.

F 4.

0	3	2	5+	3	7+	4	9+	dr̥i°	78
1	2	1	2	1	2	1	2		1

25 recto,

yutam jātam

5
2

 dvitīya guṇam

10
2

 tṛitīya ekatre

guṇitam | yutena | yutam

10
2

23
2

 yutam

33
2

 guṇitam

132
2

 riṇam jātam | pārya eśa nyāsa

5
2

5
2

23
2

123
2

 drishya 78

1

.

156
2

 vibhaktavyam

2
156

78
1

i. karaṇam | †śūnya sthāne†.....†rūpaṁ datvā†

1

 yutā jātā

5
2

 25 verso

.....

15
2

 prathamā tṛitīyasya tṛi-guṇam yutam jātam.....

chaturguṇam navārdha yutam jātam

29
2

 ekatra nyāsa.....

5
2

15
2

22
2

29
2

 dr̥i° 71

2

 prakshiptam

71
2

 bhaktam drishyam jātam

1.....anena sarvaṁ guṇitam tadeva

5
2

15
2

22
2

29
2

 ekatram ||

eshām aparo vidhiḥ ||

ii. udā° || prathama dhanam dattam najātam kim tu divardha yutam |

tadā dvitīyena dvi-guṇam dattam pañchārdha hīnam |

tadā tṛitīyena tṛiguṇam dattam saptārdha . . .

chaturtheṇa chatur-guṇam navārdha hīnam . . .

dattam ekatram ta

2	5	3	gu°	7	4	gu°	9	dr̥i	29
1	2+	1		2+	1		2+		2

F 4. [25 recto.] The example may be represented by

$$[x(1+\frac{1}{2})]+[2T_1-\frac{1}{2}x]+[3(T_1+T_2)-\frac{1}{2}x]+[4(T_1+T_2+T_3)-\frac{1}{2}x]=78.$$

Set $x=1$ and the series becomes $\frac{5}{2} + \frac{5}{2} + \frac{23}{2} + \frac{123}{2} = \frac{156}{2}$ and $\frac{78}{156/2}=1$.

[25 verso.] i. The example, of which only the solution remains, is

$$[x(1+\frac{1}{2})]+[2T_1+\frac{1}{2}x]+[3T_1+\frac{1}{2}x]+[4T_1+\frac{1}{2}x]=\frac{71}{2}, \text{ which, when } x=1, \text{ becomes } \frac{5}{2} + \frac{5}{2} + \frac{23}{2} + \frac{123}{2} = \frac{156}{2}.$$

ii. The example is $[x(1+\frac{1}{2})]+[2T_1-\frac{1}{2}x]+[3T_1-\frac{1}{2}x]+[4T_1-\frac{1}{2}x]=\frac{78}{2}$.

The solution is given on fol. 26 recto.

F 5.

- i. karanam || śūnya trūpaṁ datvāḥ yutam jātaṁ

5
2

 26 recto.
-

5
2

 prathama tṛtīyaṁ tṛi-guṇam prathamā
- chaturtham chatur-guṇam navārdha rahitaṁ | śesham

11
2

 e
- | | | | | | |
|---|---|---|----|------|----|
| 5 | 5 | 8 | 11 | dṛi° | 29 |
| 2 | 2 | 2 | 2 | | 2 |
- prakshepa yuktiḥ
- | |
|----|
| 29 |
| 2 |
- bhaktaṁ

2
29

29
2

 jātaṁ

1

 . . . gunitaṁ tad eva |
- evam riṇa rāśi bhavanti |
- ii. tṛi-prakāraṁ . . samāptaṁ || śūnya sthāne rūpaṁ datvā | tadanu
- yuktaṁ | guṇita

F 5. [29 recto.] i. This is the solution of the example given at the bottom of fol. 25 verso. Let $x=1$, then the series becomes $\frac{5}{2} + \frac{5}{2} + \frac{5}{2} + \frac{1}{2} = \frac{29}{2}$ and the correct value of x is $\frac{29}{2} \div \frac{29}{2} = 1$.

ii. "The three-fold method is completed," namely, "having put unity in the nought (empty) place; then having added . . . The śūnya sthāne rūpaṁ datvā is quoted on folios 24 verso, 25 recto and at the beginning of 26 recto.

F 6.

atha dvau		4		36	asya dalam pha°	26 verso.
athāshṭa		8		32	dalam pha°	
		16		28	dalam pha°	
4 bhu° 36		24		16	dalam pha°	
24	4	atha trīṇi usārā da				
28	4	36		20		4 asya tri
32	4	32		20		8 a
36	4	28		20		12 puna
bhu° 36		24		20		16

F 6. [26 verso.] This is, apparently, the beginning of another section, but it is isolated and although there seems to be abundance of material (compared with other leaves) I can make nothing of the problem.

G 1.

10 recto.

i. sūtram 24

ii. sūtram || *kṛitvā rūpa kshayaṁ pārtha dhānta samguṇanam tataḥ*
pravṛittir guṇanam tataḥ vinirdiset ||

iii. udā° || *ṭri-bhāga maladagdhasya ṭri-dhāntasya aiva . . .*
asṭottara-śatāni dattam kiṁ śeṣam vada paṇḍita ||

108	1	1	1
1	1	1	1
	3+	3+	3+

kṛitvā rūpa kshayaṁ pārtha† jātā 32 *śeṣa* || *prathamab† dhānte*

kshayaṁ 36 *śeṣam* 72 *dvitīyab dhānte kshayaṁ* 24 *śeṣam* 48

ṭritīyab dhānte kshayaṁ 16 *śeṣam* 32

pratyayaṁ kṛiyate | *sthāpanam*

0	1	1	1	bhā°	śeṣam	32	phalaṁ mūlā 108 atha .
1	1	1	1			1	
	3+	3+	3				

sajāti kṛiyā

G 1. Folios 10 to 15 form a fairly well defined section and the leaves are among the best preserved of the manuscript. The 'find order' is 42, 41, 40, 39, 7, 29 and the writing α2. The sūtra numbers 24 and 25 occur.

[10 recto.] i. The end of the sūtra is marked with the usual design and the sūtra is numbered 24; so that from 10 recto to the end of 15 recto consists of one sūtra (25) and its illustrative examples.

ii. Of sūtra 25 the only complete word preserved is *vinirdeset*. It is reconstructed from quotations and fragments of letters. The sūtra is the most quoted one in what remains of the original text, the phrase *kṛitvā rūpa kshayaṁ pārtha* occurring some seven times. The last word of this phrase is, however, variously written *pārtha* (fol. 10 recto), *pāstham* (10 verso), *pāstam* (12 recto et verso), *pāstha* (14 verso) and is rather curiously omitted on fol. 11 recto. This variation is very curious, because the ligatures *rtha*, *stha*, *sta* are so very unlike that the differentiation can hardly be one of carelessness in writing (and the writing is here particularly good). The meaning of the term is still obscure. Dr. Hoernle suggested *prāsta* 'thrown out' or 'wastage'; but I would translate the whole phrase by 'Having calculated for unity the loss per term.' The following is Dr. Hoernle's translation of the sūtra—

'Calculate the loss in one; let the instalments of wastage be multiplied together; with the result let the original provision be multiplied; take the result to be the required remainder.'

iii. The example may be rendered:

'The third part of the burnt bronze in three instalments (is lost). The amount given was one-hundred and eight. State the remainder, O Pandit.'

The solution according to the rule gives $108 (1-\frac{1}{3}) (1-\frac{1}{3}) (1-\frac{1}{3}) = 32$. But proceeding by steps $108 \cdot \frac{2}{3} = 72$ and the remainder is 36; $72 \cdot \frac{2}{3} = 48$ and the remainder is 24; $48 \cdot \frac{2}{3} = 32$ and the remainder is 16.

The proof may be represented by $x^3 = \frac{32}{(1-\frac{1}{3}) (1-\frac{1}{3}) (1-\frac{1}{3})}$
 Continued on the reverse;

G 1—contd.

i.	0	tribhi tryashṭa-bhāga saṃyutam	10 verso.
	1		
	1	tadāshṭottara-śatā kim	27 1 108 pha° še° 32
	3+		8 1 1
	1		
	3+	yadyekasya trayas traya aṣṭa bhāga tadā dvā-	
	1		
	3+	triṃśānām kim iti	1 3 32 phalam 108
			1 3 8 1
ii.	udā°	sakṛid dhāntasya lohasya daśāṃśhā kshīyate-s-trayam	
		saptate dviguṇā . cha kim śesham vada paṇḍitah	3 140
			10
		†kṛitvā rūpa kshayam pāstham† iti . . rūpam	1 3
			10
	jātam śesha	7 mūlam 140 anena guṇitam jātam 98 kshayam 42	10 1
	evam	140	
		7 1 98 phalam 140	10 1 1

G 1. [10 verso.] i. Gives further proofs of the example on the obverse, namely:

$$x^1 (1 - \frac{1}{10}) (1 - \frac{1}{10}) (1 - \frac{1}{10}) = <32, \text{ hence } x=108>;$$

then two proportions in words and figures $\frac{27}{8} : 1 :: 108 : 32$ and $1 : 3\frac{3}{8} :: 32 : 108$.

ii. Example.—Of iron once refined three-tenths is lost. What is the remainder of twice seventy, tell me Pandit ?

The loss on unity is $\frac{3}{10}$ and the remainder is $\frac{7}{10}$. The original quantity is 140 and $\frac{7}{10}$ of 140=98. The loss is therefore 42 and 98+42=140.

Proof. $\frac{7}{10} : 1 :: 98 : 140$

Continued on fol. 11 recto.

G 2.

i.	pratyayah	0		11 recto.
		1		
		1		
		3+		
		10		
ii.	udā°		palā krīte pala tri-bhāgam kshya vrajati	
			aṣṭā-daśa thatām brūhi	

G 2. [11 recto.] i. Continued from fol. 10 verso. 'Proof $x(1 - \frac{3}{10}) = <98, \text{ therefore } x=140>$

ii. Example.—In purchasing one and a half *palas* the loss is one-third. State what would be the loss on eighteen.

Since $\frac{1}{3} / \frac{3}{2} = \frac{2}{3}$, the loss on unity, the remainder is $\frac{1}{3}$. Now $\frac{2}{3}$ of 18 =14 and the loss is 4.

Proof by the rule of three :— $1\frac{1}{2} : \frac{1}{3} :: 13 : 4$ and $\frac{1}{3} : 1\frac{1}{2} :: 4 : 18$.

G 2—contd.

1	3	bhā	18
3	2		1

karanam | addhyardha palam-s-chhedebhya idam

2
9

†kṛtvā rūpa

kshayam† rūpam

1

kshayam kṛtvā jātam

7	18
9	1

guṇitam jātam

14	kshayam	4
----	---------	---

pratyaya trai-rāśikena ||

addhyardha pala kṛite

tṛi-bhāgam kshaya gachchhati |

ashtā-daśa pala kṛita

kiṁ kshayam vada paṇḍita ||

1	1	18	phalam	4
1	3	1		1
2				

puna tṛi-bhāga divardham tadā chatubhi x kiṁ iti

1	1	4	phalam	18
3	1	1		1
	2			

iii. udā° || chatur-bhāga mala dagdha suvarṇa śata-paṁchakam |

. atha pratyay 11 verso

0	158	su°	phalam mūla	500		punar eva prastāra kṛamam
1	1	to°				
1	5*					
4+	1		500	1	1	1
1	64		1	1	1	1
4+			4+	4+	4+	4+
1						
4+						
1						
4+						

śesha 158 to° 1 še° 1
64

iii. Example.—In refining bronze there is a loss of one-fourth. What would be the loss on 500 suvarṇas four times refined?

The solution is lost. It amounted to $< 500 (1-\frac{1}{4}) (1-\frac{1}{4}) (1-\frac{1}{4}) (1-\frac{1}{4}) = 158\frac{1}{8} = 158 \text{ suvarṇas} + 1\frac{1}{8} \text{ tolās}$, since $5 \text{ tolās} = 1 \text{ suvarṇa}$.

Continued on the reverse.

G 2. [11 verso.] This appears to have contained five proofs of the example on the obverse, for the present third proof is designated 'the fourth.' The proofs are—

i. Missing.

ii. $x^4(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4}) = 158 \text{ su}^\circ + 1\frac{1}{8} \text{ to}^\circ$ therefore $x^4 = 500$.iii. $500 (1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4}) = x^4$ and $x = 158 \text{ su}^\circ + 1\frac{1}{8} \text{ to}^\circ$.iv. $x^4 = (158 \text{ su}^\circ + 1\frac{1}{8} \text{ to}^\circ) \div (1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})$ and $x^4 = 500$.v. The first loss is $\frac{60}{4} = 125$ and the remainder is 375.The second loss is $\frac{375}{4} = 93\frac{3}{4} = 93 \text{ su}^\circ + 3 \text{ to}^\circ + 9 \text{ māsha}$. (Since $12 \text{ mā}^\circ = 1 \text{ to}^\circ$) and the remainder is $281\frac{1}{4}$.The third loss is $281\frac{1}{4} \div 4 = 70\frac{3}{8}$ add the remainder is $210\frac{1}{8}$.The fourth loss is $210\frac{1}{8} \div 4 = 52\frac{3}{8}$ and the remainder is $158\frac{1}{8}$.

G 2—concl'd.

anyam chaturtha pratyayam kriyate

0	1	1	1	1	bhā°	śesha.	158	phalam	500	
1	1	1	1	1			1			
	4+	4+	4+	4			5*			
							1			
							64			

ādyam kshayam 125 śesham 375 | dvitiye kshayam 93 to° 3 māsa 9

śesham	281	kshayam	70	śesham	210	kshayam	52
	1		5		15		47
	4		16		16		64

śesham	158	eśa sarvatra kartavyā	
	13		
	64		

G 3.

12 recto.

. prastha madhunās tathāh

ambhasa

†kṛitvā rūpa kshayam pāstam† iti : tatra kshayam : pāstam : iti : tatra kshaya :

rūpam guṇya śesham	3	3	3	3	4	gadyūti gadyūti gatvā-
	4	4	4	4	1	

t-prastham pivot gadyūti yojanam | chatu prasthai

ādhakam tadā dhāntaśor gu . . tatah	81	āvṛitti pravṛittir-guṇanam tatah
	256	

4 anena guṇitam jātam	81	eśa maddhva bhāgā bhāge hṛite labdham
	64	

1 madhu prastha 1 kū° 1 še°	1	ambha bhāgā prastha 2 kūḍava 2
	16	

še	15	evam	4	. . . kūḍavokti prakshepake ādhakā śoḍashā kūḍavā
	16			

bhavanti | 16 | ato ma śesham 12

G 3. [12 recto.] This is not directly connected with folio 11 but is probably correctly placed here. The find order places it between folios 11 and 13 and it is definitely connected with folio 13. Also it quotes from *sūtra* 25 on folio 10 recto. It has the same knot as folio 13.

The example may be conjecturally restored : A traveller goes a journey of 4 *gavyūti*s and takes with him 4 *prasthas* of wine. After each *gavyūti* he drinks 1 *prastha* and then fills up his bottle with water. How much wine and how much water will there be at the end of his journey ?

The preliminary part of the solution is rather confused. Possibly the *visarga* marks denote deletion. The general solution is $4 \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = 4 \cdot \frac{27}{64} = 2\frac{13}{16} = 1\frac{13}{8}$ *prasthas* of wine remain and $2\frac{13}{16}$ *prasthas* of water. The number of *gavyūti*s in a *yojana* are mentioned (12), and the number of *prasthas* in an *ādhaka* are said to be 4 and the number of *kūḍavas* in an *ādhaka* are given as 16. Therefore the wine left over $2\frac{13}{16} = 1$ *prastha* + $1\frac{13}{16}$ *kūḍavas* and the water $= 2\frac{13}{16} = 2$ *prasthas* + $2\frac{13}{16}$ *kūḍavas* and the sum of these is 4 *prasthas*.

Continued on the reverse.

G 3—contd.

i. prastha kuḍavā | 4 | 3 | śesha chatvāra . . . 12 verso.

kuḍavaḥ	2 1 4	2 1 4	śeshā cha kuḍavā pītā ma°	7 1 4 ⁺	9 1 4	puna
---------	-------------	-------------	-----------------------------	--------------------------	-------------	------

chatvāri kuḍavā bhuktaṁ śesham	81 16	175 16	jala bhāgaṁ madhu kudava
--------------------------------	----------	-----------	----------------------------

5 1	śe°	1 16	jala kuḍava	10 1	śe°	15 16	evaṁ kuḍava 16
--------	-----	---------	-------------	---------	-----	----------	----------------

ii. udā° || datvā śulkaṁ chatur bhāgaṁ aṣṭau āṇita kuṅkumā |

chatu śulka śālais tu kiṁ śesham vada paṇḍita :||

8 1 4 ⁺

karanam		†kritvā rūpa kshayaṁ pāstaṁ† pāstaṁ	8 1	3 4	guṇitaṁ
---------	--	-------------------------------------	--------	--------	---------

jātaṁ	6	śulke	2	śesham	6 1 1 4 ⁺	anena guṇitaṁ jātaṁ
-------	---	-------	---	--------	-------------------------------	---------------------

4 1 2	kshayaṁ	1 1 2	śeshena	4 1 2	1 1 4 ⁺	datvā guṇita jāta	27 8
-------------	---------	-------------	---------	-------------	--------------------------	-------------------	---------

.

G 3. [12 verso.] i. The solution of the example on the obverse is now done by steps. The original amount of 4 *prasthas* is expressed in *kuḍavas*, namely 16.

Of these 16 *kuḍavas* of wine he drinks $\frac{1}{4}$ and 12 are left and he adds 4 of water. He then drinks $\frac{1}{2}$ of wine and there are 9 *kuḍavas* left and the water is made up to 7 *kuḍavas*. Then he consumes $\frac{2}{3} = 2\frac{1}{3}$ of wine and there are $9 - 2\frac{1}{3} = 7 - \frac{1}{3}$ and the water is made up to $9\frac{1}{3}$. He then drinks $\frac{61}{4} = 3\frac{3}{4}$ and there is left $6\frac{1}{3} - 3\frac{3}{4} = \frac{5}{12}$ and the water is made up to $1\frac{1}{6}$. There is, therefore, finally $\frac{5}{12} = 5\frac{1}{6}$ *kuḍavas* of wine and $1\frac{1}{6} = 10\frac{1}{6}$ *kuḍavas* of water and these added together give 16 *kuḍavas*. See part I, § 89.

ii. *Example*.—Having given one-quarter as toll at four toll-houses eight of saffron is brought in. State, O Pandit, what is left.

Solution. $8 \times \frac{1}{4} = 2$ and 2 is paid in toll; $6 (1 - \frac{1}{4}) = 4\frac{1}{2}$ and the loss is $1\frac{1}{2}$; $4\frac{1}{2} (1 - \frac{1}{4}) = 3\frac{3}{4}$ and the toll is $1\frac{1}{2}$; $3\frac{3}{4} (1 - \frac{1}{4}) = 2\frac{7}{8}$ and the last toll is $\frac{7}{8}$; and the total toll paid is $2 + 1\frac{1}{2} + 1\frac{1}{2} + \frac{7}{8} = 5\frac{5}{8}$ which leaves $8 - 5\frac{5}{8} = 2\frac{3}{8}$.

Continued on fol. 13 *recto*.

G 4.

13 recto.

i.	<table> <tr> <td>8</td><td>1</td><td>1</td><td>1</td><td>1</td></tr> <tr> <td>1</td><td>$\frac{1}{4}+$</td><td>$\frac{1}{4}+$</td><td>$\frac{1}{4}+$</td><td>$\frac{1}{4}+$</td></tr> </table>	8	1	1	1	1	1	$\frac{1}{4}+$	$\frac{1}{4}+$	$\frac{1}{4}+$	$\frac{1}{4}+$	guṇitaṁ jātaṁ	<table> <tr> <td>81</td></tr> <tr> <td>32</td></tr> </table>	81	32	punānyaṁ
8	1	1	1	1												
1	$\frac{1}{4}+$	$\frac{1}{4}+$	$\frac{1}{4}+$	$\frac{1}{4}+$												
81																
32																

<table> <tr> <td>8</td><td>3</td><td>3</td><td>3</td><td>3</td></tr> <tr> <td></td><td>4</td><td>4</td><td>4</td><td>4</td></tr> </table>	8	3	3	3	3		4	4	4	4	phalaṁ	<table> <tr> <td>81</td></tr> <tr> <td>32</td></tr> </table>	81	32	punānyaṁ	<table> <tr> <td>8</td></tr> <tr> <td>1</td></tr> <tr> <td>$\frac{1}{4}+$</td></tr> <tr> <td>$\frac{1}{4}+$</td></tr> <tr> <td>$\frac{1}{4}+$</td></tr> <tr> <td>$\frac{1}{4}+$</td></tr> <tr> <td>$\frac{1}{4}+$</td></tr> <tr> <td>$\frac{1}{4}+$</td></tr> </table>	8	1	$\frac{1}{4}+$	$\frac{1}{4}+$	$\frac{1}{4}+$	$\frac{1}{4}+$	$\frac{1}{4}+$	$\frac{1}{4}+$	phalaṁ
8	3	3	3	3																					
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<table> <tr> <td>81</td></tr> <tr> <td>32</td></tr> </table>	81	32	puna pratyayaṁ	<table> <tr> <td>0</td></tr> <tr> <td>1 bhā</td></tr> <tr> <td>1</td></tr> <tr> <td>$\frac{1}{4}+$</td></tr> <tr> <td>1</td></tr> <tr> <td>$\frac{1}{4}+$</td></tr> <tr> <td>1</td></tr> <tr> <td>$\frac{1}{4}+$</td></tr> <tr> <td>1</td></tr> <tr> <td>$\frac{1}{4}+$</td></tr> </table>	0	1 bhā	1	$\frac{1}{4}+$	1	$\frac{1}{4}+$	1	$\frac{1}{4}+$	1	$\frac{1}{4}+$	phalaṁ kuṁkuma	8
81																
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ii. udā° || tri-bhāga śaḍ-bhāga pañchāhśam . guḍapiṇḍ āsṭabhāraḥ |
 kiṁ śeṣaṁ dattabhir bhavet || . . .

<table> <tr> <td>8</td><td>2</td><td>5</td><td>4</td></tr> <tr> <td>1</td><td>3</td><td>6</td><td>5</td></tr> </table>	8	2	5	4	1	3	6	5	guṇitaṁ jātaṁ	<table> <tr> <td>32</td></tr> <tr> <td>9</td></tr> </table>	32	9	etat phalaṁ
8	2	5	4										
1	3	6	5										
32													
9													

iii. udā° || chatu ḥ pañchaka lābhena daśa droṇāt prayojita |
 tad vai tribhis tu kiṁ lābhaṁ katthyatām gaṇakottama |||

<table> <tr> <td>10</td><td>5</td><td>5</td><td>5</td></tr> <tr> <td>1</td><td>4</td><td>4</td><td>4</td></tr> </table>	10	5	5	5	1	4	4	4	guṇitaṁ jātaṁ	<table> <tr> <td>1250</td></tr> <tr> <td>64</td></tr> </table>	1250	64
10	5	5	5									
1	4	4	4									
1250												
64												

G 4.

[13 recto.] i. Here are four 'proofs' of the example given on folio 12 verso.

(a) $8(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})=\frac{3}{4}$.

(b) $8\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}=\frac{3}{4}$.

(c) $8(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})=\frac{3}{4}$.

(d) $x^4=\frac{81}{32}(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})(1-\frac{1}{4})$, whence $x^4=8$.

ii. Example.—There is a quantity of molasses weighing eight *bhāras*. What will be left after giving away one-third, one-sixth and one-fifth?

$8\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{5}{6}=\frac{5}{2}$ and this is the answer.

iii. Example.—By a gain of five-fourths ten *droṇas* are obtained. Let it be said, O best of calculators, what will be the gain by three transactions.(Here the term *lābha* seems to have meaning 'capital + profit,' what is termed the 'mixed quantity' *miśra* on folio 62.)

$10\cdot\frac{5}{4}\cdot\frac{5}{4}\cdot\frac{5}{4}=\frac{1250}{64}<=19\frac{17}{32}=19\text{ dro}^\circ+2\text{ ād}^\circ+0\text{ pra}^\circ+2\text{ ku}^\circ>$

For these measures see part I, §109.

Continued on the reverse.

G 4—contd.

i.	0	bhā°	śe° 19	phalaṁ 10		0	phalaṁ dro° 19 ā° 13 verso.
	1		1			1	
	1		ā° 2			1	2 pra° 0 ku° 2
	4		4* dro°			4	
	1		pra° 0			1	. . .
	4		4* ā° pra°			4	
	1		ku° 2			1	
	4		ku° 4* prasthi			4	

ii. udā° || kasyāpyarjjakasya shashṭhi sva-dalena kshayaṁ gata |
 puna vṛiddhyā tri-bhāgena sva-pādena tatojjhitam
 vṛiddhyā tu pañcha-bhāgenas tathā vṛiddhi dvayo gatam |
 kā vṛiddhi syā kim vā śeshaṁ tad uchyatām ||

60	1	1	1	1	rūpa lā . . . jātā 36	
1	1	1	1	1		
	2+	3	4+	5		

pratyayaṁ punasyaiva	0	1	1	1	1	bhā°	36	phalaṁ 60	
	1	1	1	1	1		1		
		2+	3	4+	5				

punānyaṁ pratyayaṁ	60	phalaṁ 36		mūlaṁ na jñāyate
	1				
	1				
	2+				
	1				
	3				
	1				
	4+				
	1				
	5				

0	1	1	2	1	3	1	4	1	5
---	---	---	---	---	---	---	---	---	---

phalaṁ . . .

G 4. [13 verso.] i. Continued from the obverse.

$$(a) \quad x^1 = \frac{19 \text{ dro}^\circ + 2 \text{ ā}^\circ + 0 \text{ pra}^\circ + 2 \text{ ku}^\circ}{(1+\frac{1}{4})(1+\frac{1}{3})(1+\frac{1}{2})} = 10.$$

$$(b) \quad x^1 (1+\frac{1}{4})(1+\frac{1}{3})(1+\frac{1}{2}) = 19 \text{ dro}^\circ + 2 \text{ ā}^\circ + 0 \text{ pra}^\circ + 2 \text{ ku}^\circ < \text{whence } x^1 = 10 >. \text{ See Part I, p. 62.}$$

ii. Example.—The capital of a certain banker is sixty. One half of it goes in loss and then he gains by one-third; next he loses one-fourth of it and finally gains one-fifth; so that he has two gains. What is his gain and what is his loss and what the remainder and let that be stated.

$$\text{Solution: } 60 (1-\frac{1}{2})(1+\frac{1}{3})(1-\frac{1}{4})(1+\frac{1}{5}) = 36.$$

$$\text{Proofs. (a) } x^1 = \frac{36}{(1-\frac{1}{2})(1+\frac{1}{3})(1-\frac{1}{4})(1+\frac{1}{5})}, \text{ whence } x^1 = 60.$$

$$(b) \quad 60 (1-\frac{1}{2})(1+\frac{1}{3})(1-\frac{1}{4})(1+\frac{1}{5}) = 36$$

$$(c) \quad x^1 (1-\frac{1}{2})(1+\frac{1}{3})(1-\frac{1}{4})(1+\frac{1}{5}) = 36 < \text{whence } x^1 = 60 >$$

G 5.

14 recto.

yasya tanmayatā chakshu

1	1	1
3	4	5

 apahṛita śulka piṇḍam 24 ||

 karaṇam || †kṛitvā rūpā kshayaṁ pāsta†

2	3	4
3	4	5

 jātu saṁgunya

 jātam

2
5

 etāvad api rūpa saṁśudhā jātam

3
5

 anena bhaktvā śulka

piṇḍam guṇitam jātam 40 eśa piṇḍam

 pratyayaṁ

2	40
5	1

 guṇita jātam 16 śeṣam 24 evam 40

 anyam asya pratyayaṁ

40
1
1
3+
1
4+
1
5+

 phalam 16 kshayaṁ 24 evam 40 ||

ii. udā° || guda piṇḍa jñāta tulyoś chatu . . . avye guḍam |

tri-chatu φ -pañcha-shaḍ vṛiddhyā chatvāriṁśa (bha*) ve kshaya .

 G 5. [14 recto.] i. The find order of folio 14 is unknown. It introduces a variation of the problems given on folios 10 to 13, but it still quotes from the same sūtra or a very similar one. The first example can be represented by $x(1-\frac{1}{3})(1-\frac{1}{4})(1-\frac{1}{5})=x-24$.

 Solution: $\frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{2}{5}$, $1 - \frac{2}{5} = \frac{3}{5}$, $\frac{3}{5} \cdot 40 = 24$ and this is the quantity (piṇḍam).

 Proof: $\frac{2}{5}$ of 40 = 16 and 40 - 16 = 24.

 Another proof of this: $40(1-\frac{1}{3})(1-\frac{1}{4})(1-\frac{1}{5}) = 16$ and 40 - 16 = 24.

ii. Example.—A known amount of molasses equal to . . . four is increased by one-third, one-fourth, one-fifth, one-sixth and then forty is lost . . .

No solution is preserved.

i. udā° || ajñātārambha-lohasya tri-chatu φ -pañchakā kshaye |

14 verso.

sapta-vimśati piṇḍasya tri-dhānta śeṣhya dṛishyate |

kiṁ sarvaṁ vada tatvajña kshayaṁ cha mama katthyatām ||

1	1	1	śe°	27
3	4	5		1

G 5. [14 verso.] (i) Example.—An unknown quantity of lapis-lazuli loses one-third, one-fourth, and one-fifth; and the remainder after the three-fold operation on the original quantity is twenty-seven. State what the total was, O wise one, and also tell me the loss.

 Solution $\frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{2}{5}$; $1 - \frac{2}{5} = \frac{3}{5}$; $27 \div \frac{3}{5} = 45$ and 45 - 27 = 18 and this is the loss.

The meaning of ambha-loha = lapis-lazuli was suggested by Dr. Hoernle.

G 5—contd.

karaṇaṁ | †kritvā rūpa kshayaṁ pāstha†

2	3	4
3	4	5

 guṇitaṁ

jātaṁ

2
5

 rūpa kshayaṁ

3
5

 anena śeṣaṁ bhaktaṁ śeṣaṁ

27

bhaktaṁ jātaṁ 45 aśya sapta-vimśa | pātya śeṣaṁ 18 | eta
kshayaṁ ||

iii. udā° :|| parikṣhīṇasya lohasya tri-dhāntaṁ pañcha māśakaṁ |
na jñāyatet pravṛittkāṁ na tu śeṣa pradṛisyate |
pravṛitti śeṣaṁ yo piṇḍaṁ kevalaṁ vimśati sthitaṁ |
ajñāta kām pravṛitti syā kim vā śeṣaṁ vadaśva me ||

1	1	1
3	4	5

 kritvā

ii. *Example*.—Of the loss of iron the third is one-fifth of a māsha. The original quantity is not known and neither is the remainder given; but only the original remainder which quantity stands at twenty. Tell me what is the unknown original quantity and what is the remainder.

This interpretation, however, is by no means certain. The solution is lost.

G 6.

. pravṛitti bhavet sakhe ||

15 verso

1	1	1	1
3	3	3	3

 śe 16
1

karaṇaṁ :|| dhāntaśo ghātitaṁ tena | †rūpa kshayaṁ kritvā† jātaṁ

G 6. [15 verso.] There is a suspicion that this is a double leaf. The lenticels on the left side are well-marked but hardly any trace of them appears on the right side. Also the contents are to some extent incongruous.

The example may be represented by $x(1-\frac{1}{3})(1-\frac{1}{3})(1-\frac{1}{3})(1-\frac{1}{3})=16$. Now $\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{16}{81}$ and $16 \div \frac{16}{81} = 81$ and this is the original quantity.

Another method by *kala-savarna*. (This term laterally means 'parts resembling one-sixteenth,' but by Mahāvira it is used to denote fractions generally iii. 1). The question is inverted: 'Of iron (refined) four times eighty-one is given. What is the remainder, state, O expert, which is solved by working hard in calculating.'

$81(1-\frac{1}{3})(1-\frac{1}{3})(1-\frac{1}{3})(1-\frac{1}{3})=16$.

"Another proof is made and the original amount is not known."

$$x^1 = \frac{16}{(1-\frac{1}{3})(1-\frac{1}{3})(1-\frac{1}{3})(1-\frac{1}{3})} = 81 \text{ pala.}$$

G 6—contd.

2	2	2	2
3	3	3	3

 guṇitam

16
81

 bhaktam

81
16

 śeshena guṇaye

śesham

16
1

 guṇita jātā

81
1

 $\dots \text{pravṛttir ity arthaḥ} \quad || \quad \text{athānya}$

vidhi kalā savarṇe

chatur dhānta . lohasya ekāśītis-cha dattavān

kim śesham vada dharmajña ya gaṇite kritam śramam ||

81	1	1	1	1
1	$\frac{1}{3}+$	$\frac{1}{3}+$	$\frac{1}{3}+$	$\frac{1}{3}+$

 phalam śe° 16 ||

puna pratyayam kṛiyate mūlam na jñāyate

0	1	1	1	1	bhā°	śe°	16
1	$\frac{1}{3}+$	$\frac{1}{3}+$	$\frac{1}{3}+$	$\frac{1}{3}+$			1

 phalam loha pala 81 ||

udā°

15 recto.

. kaśchi yadi śakya tad uchyatām ||

etan me saṁśayam prājñad dhānta kshayam vichāraṇāḥ

2	3	4
3	4	5

 ksha° śe° 32
1

$\text{karaṇam} \quad || \quad \text{dhānta saṁguṇya guṇitam jātām}$

3
5

 rūpam dadyā

8
5

$\text{bhāge hṛite labdham bhak} \dots \dots \dots$

5	32
8	1

 $\text{phalam 20 eśa sā pravṛitti}$ |

$\text{śesham 12} \dots \dots \dots 32 \quad || \quad \text{pañcha-vimśatima sūtram} \quad || \quad 25$

[15 recto.] Only the end of the formal question is preserved—If thou canst state . . . this is my doubt, O wise man, by examination

The example may have been:— $(1-\frac{1}{3})(1-\frac{1}{4})(1-\frac{1}{5})=x-r$ and $x+r=32$. From this $\frac{2}{3}x=x-r$, $(1-\frac{1}{3})x=r$, $\frac{2}{3}x+x=32$ and $x=32 \div \frac{5}{3}=20$, and $r=\frac{2}{3}.20=12$.

G 7.

i. vibhaktam jātam

2	śe°	10
9		1

9
7

 16 recto.

anena guṇitam jātam

90
7

 bhāge hṛite labdham 12 ||

asya pratyaya trai-rāśikena

7	1	10	pha°	12
6	1	1		6
	2			7

ii. udā° || mākshikag-ghaṭakasyaiva dvi-tri-bhāga pravardhitam
 dvitiye dvi-paṁchamo-bhāgo tritiye dvi-saptakoḍbhavam
 chaturthe dvi-navam-bhāgam evam jāta pala trayam |
 babhūvā saulkikai hṛitvā kim sarvam vada paṇḍita ||

2	2	2	2	śe°	3
3	5	7	9		1

dhāntaso iti | kṛitvā

- G 7. [16 recto.] i. The find order is 30 and the writing is α2,4. Only the remnants of a problem: Loss on $1\frac{1}{2}$ is $7/6$; what is the original when the remainder is 10? Loss on 1 is $\frac{7}{6} \div 1\frac{1}{2} = \frac{7}{9}$ therefore $x \cdot \frac{7}{9} = 10$ and $x = \frac{90}{7} = 12\frac{6}{7}$.
 Proof by the rule of three: $\therefore 1\frac{1}{2} : : 10 : 12\frac{6}{7}$.
 ii. Example.—Of a *ghaṭakā* of honey two-thirds is given, to the second two-fifths, to the third two-sevenths, to the fourth two-ninths, till only three *palas* (are left). O Pandit, state how much altogether was taken away by the tax collector.

H 1.

sūtram

16 verso.

- i. idāni suvarṇa kshayaṁ vakshyāmi . . syedaṁ
- ii. sūtram || kshayaṁ saṁguṇya kanakās tadyutir bhājayet tataḥ
saṁyutair eva kanakair ekaikasya kshayo hi saḥ
- iii. udā° || eka-dvi-tri-chatus saṁkhyā suvarṇā māśakai riṇai |
eka-dvi-tri-chatus saṁkhyā rahitā sama-bhāgatām ||

sthāpanaṁ kṛiyate | eshām

1+	2+	3+	4+
1	2	3	4

karaṇaṁ || †kshayaṁ saṁguṇya kanakādibhi† kshayena saṁguṇya jātaṁ
| 1 | 4 | 9 | 16 | ... | esha yuti 30 | kanakā yuti 10 anena
bhaktvā labdham

- H 1. [16 verso.] i. The end of a sūtra is marked but the number is not preserved (probably 26) and then a new section is introduced by the remark—"Now I shall speak about suvarṇa kshaya." It should be noted that Mahāvira uses the term kshaya as synonymous with varṇa in his section (vi, 169ff) on suvarṇa kṣīṭikāra. In our text there seems to be some confusion about the meaning of kshaya which here really means varṇa or 'quality,' although the author obviously thought it denoted a loss. Mahāvira's rule is—

Kanaka kshaya saṁvargo mitrasavarṇāhṛitaḥ kshaya jñeyah |

paravarṇa pravibhaktāni suvarṇa guṇitāni phalaṁ hemnaḥ || 169 ||

"It should be known that the products of gold kshaya, when divided by the mixed gold gives rise to the kshaya. When divided by the last varṇa (=kshaya) and multiplied by the gold gives the corresponding quantity of gold."

ii. Rule.—Having multiplied the parts of gold with the kshaya let this sum be divided by the sum of the parts of gold. The result is the average kshaya. This means $f = \frac{f_1 g_1 + f_2 g_2 + f_3 g_3 + \dots + f_n g_n}{g_1 + g_2 + \dots + g_n}$ where f denotes kshaya and g gold.

iii. Example.— $f_1=1, f_2=2, f_3=3, f_4=4$ and $g_1=1, g_2=2, g_3=3, g_4=4$ therefore $f = \frac{1.1+2.2+3.3+4.4}{1+2+3+4} = \frac{30}{10} = 3$.
Continued on fol. 17 recto.

H 2.

- i.
- | | | | | | |
|----|----|---|------|-----|--------|
| 1 | 1 | | | | |
| 10 | 30 | 4 | pha° | mā° | śe° 12 |
| 1 | 1 | 1 | | | 1 |

17 recto.

- ii. udā° || eka-dvi-tri-chatus saṁkhyā suvarṇa projjhitā ime
māśakā dvi tṛitām chaiva chatu saṁkhyā pañchakarāṁśakāṁ
kiṁ kshayaṁ

1	2	3	4
1	1	1	1
2	3	4	5

- H 2. [17 recto.] i. The remnant of a proof of the example given on 16 verso.

10 : 30 :: 4 : 12, i.e., $\Sigma g : \Sigma fg :: g_r : g_r F$.

ii. Example.—Gold one, two, three, four; 'abandoned' the following māśakas one-half, one-third, one-fourth and one-fifth.

$$F = \frac{1.1+2.2+3.3+4.4}{1+2+3+4} = \frac{30}{10} = 3$$

'Proof by the rule of three' $\Sigma g : \Sigma fg :: g_r : g_r F$.

H 2—contd.

karaṇam || †kshayaṁ saṁguṇya kanakā† eśa sthāpayate |

1	2	3	4
2	3	4	5

†tad yutir bhājayet tataḥ† hara sāsyē kṛite yutam

163
60

† saṁyutai x

kanakair† bhaktvā tadā kanaka

10

anena bhaktam jātam

163
600

eśa

ekaika suvarṇasya kshayaṁ ||

pratyaya trai-rāśikena . . .

10	163	1	pha°	163
1	60	1		600
10	163	2	pha°	163
1	60	1		300
10	163	3	pha°	163
1	60	1		200
10	163	4	pha°	163
1	60	1		150

17 verso.

krameṇa dvaya māśhādi uttare eka hīnatām |

suvarṇam me tu sammiśrya katthyatām gaṇakottama ||

sthāpaṇam	4+	5+	6+	7+	8+	9+	1+	2+	3+
	5	6	7	8	9	10	2	3	4

†kshayaṁ saṁguṇya† jātam | 20 | 30 | 42 | 56 | 72 | 90 | 2 |

6 | 12* | eśām yuti | 330 || kanakānām yuti 45 | anena bhaktvā

labdham | 330 | pañcha-daśa bhāge chchheda kṛiyate | phalam | 7 śe° 1 |

45

1

3

eśaa ekaika māśaka kshayaṁ |

pratyaya trai-rāśikena

45	330	1	phalam	22
1	1	1		3

evam sarveshām pratyayam

H 2. [17 verso.] I do not understand the problem but it is explained by Dr. Hoernle in the *Indian Antiquary* of 1888 (Vol. XVII, p. 43).

The solution is $F = \frac{5.4+6.5+7.6+8.7+9.8+10.9+2.1+3.2+4.3}{5+6+7+8+9+10} = \frac{330}{45} = 7\frac{1}{3}$.

Proof by the rule of three— 45 : 330 :: 1 : $7\frac{1}{3}$ and 'so for all of them.'

* Inadvertently omitted in the manuscript.

H 3.

i. (sūtram) || aprāpta saṁguṇā kaṭi kāmchanāni tatojjhitam

18 recto.

kāmchanai yad bhava labdha sa kshaya jñāta māśaka ||

ii. udā° || eka-dvi māshako prāpto dvau cha prāptam cha pañchabhi |

trayaś cha katibhiḥ prāpta shaḍ eva . ni kevalam |

chaturbhi māshakair hīnam kaṭi dṛishṭvā mayā sakhe |

trayaś cha katibhiḥ prāptā suvarṇam māśako vadaḥ |

1	2	3	6
2	5	0	4+

karaṇam || †aprāpta saṁguṇā kaṭid† iti

6
1

 aprāpta kaṭi chatvāra

4

saṁguṇya jātam

24

 †kāmchanāni tatojjhitam† dvābhyām eka pañchabhi

dvayam saṁguṇya jātam 2

10
1

 tad yuti 12 | hitvā 2

hitvā jātam śeṣam 12 || aprāpta gaṇḍikai

H 3. [18 recto.] i. The sūtra is largely restored from the quotations given in the solution below. The application of the terms *aprāpta* and *kaṭi* are not at all clear; but given that

$$F = \frac{f_1 g_1 + f_2 g_2 + f_3 x}{g_1 + g_2 + x} \text{ then the sūtra states that } x = \frac{F \cdot \Sigma g - (f_1 g_1 + f_2 g_2)}{f_3}$$

ii. *Example.*—Māshakas of one and two, gold of two and five, māshakas of three and gold unknown. All that is known is the sum of māshakas, six; and the average māshaka four. State the māshaka of the unknown gold.

Statement $f_1=1, f_2=2, f_3=3; g_1=2, g_2=5, g_3=x; F=4.$

$$\text{Solution } x = \frac{4 \cdot 6 - (2 \cdot 1 + 5 \cdot 2)}{3} = \frac{24 - 12}{3} = \frac{12}{3} = 4 \dots \dots$$

. ashta-vimśatima sūtram

18 verso.

i. sūtram || ūnais saṁguṇya kanakā tat piṇḍam cha viśodhayet

suvarṇa kanakābhyastā rāśi shesham vibhājayet

aprāpta gaṇḍika śeṣa śuddhena kanakena tu |

yal labdham tat pramāṇam tu gaṇḍikā yā vinirdiset ||

H 3. [18 verso.] The end of the 28th sūtra is marked.

i. *Rule.*—Having multiplied together the (known) gold pieces and their *varṇas* determine the sum of that. Divide the remainder of that quantity and the sum of the product of the average *varṇa* and known gold by the difference between the average *varṇa* and the *varṇa* of the unknown gold. That which results consider to be the measure of the unknown gold.

$$F = \frac{f_1 g_1 + f_2 g_2 + f_3 x}{g_1 + g_2 + x} \text{ then } x = \frac{(f_1 g_1 + f_2 g_2) - F (g_1 + g_2)}{f_3}$$

H 3—*contd.*

- ii. udā° || eka-dvi-¹tri-chatus saṁkhyā aprāpta māśakāni tu
 eka-dvi-¹tri-chatus saṁkhyā ekatrāvartitā kilah
 gaṇḍikā jñāta kanakā ūnaikā daśa māśakai |
 aprāpta jñāta kanakai pra yah

1	2	3	4	0
1	2	.	.	.

karaṇam

J 1.

30 recto.

sūtram | eka yuta nara sarvash shaḍbhi pa

. anena labdham hītā pratham

36	42	48	54	6
..	..	78	7.	

. sadṛiśa kṛi bhāga hāram kṛiyate

234
70

 30 verso.

. . . . tulāḍhe

3
24
70

 mudgāḍhe

1
47
..

 kṛiyate

J 2. [Folio 30.] Find order 32. Writing α4. By appearance this fragment and fol. 28 perhaps belong to the same leaf. See also fol. 31.

[30 recto.] A restoration is suggested in *part I*, §78, vii, but I doubt its being correct.

[30 verso.] We have $\frac{2\frac{1}{2}}{70} = 3\frac{1}{2}$ and $3\frac{1}{2} \div 2 = 1\frac{1}{2}$. The term *mudga* ? 'a kidney bean' occurs also on folio 31. See also *Līlāvati*, §97.

J 2.

etat-kāla timanushyā ya lagyanti 65 recto.

apara prashṇaḥ

yady eka purushasya drammāsh-shaṭ. *triṃśabhir* dinai jīva-lokā | tat kāryam

prastutam . ssaptatīnām pāka rākshakānām drammaish-shaḍbhi

kati dinā jīva-lokam bhavati . . .

karaṇam | ādau tāva yady ekapurushasya drammāsh-shaṭ *triṃśabhi* - - -

jīvyāḥ | tat saptatīnām kim

1	pu°	dram°	6	30	di°	70	pu°	phalam
1			1			1			

drammā *triṇi* śata-sā

J 2. [65 recto.] Folio 65 consists of two leaves stuck together. The *verso* side has been definitely placed as C 4. The writing is here α4. The find order is unknown.

[Example :—If a man requires six drammās for his livelihood for 30 days, for how many days will 70 men (guards of a fort?) live on six drammās ? The details are, however, uncertain.—K. N. D.]

J 3.

. . . . dramṃā asṭa dvā-chatvālimśabhir dinai | tat saptati 41 recto.

ya 42 1	dine	dram ^o 8 1	jīvyā 70 purushā 42 1 1
------------	------	--------------------------	------------------------------

dramṃā 560 || yadi pañcha-śata-śasṭyādhika dva-chatvālimśabhi
tad drammai asṭabhi kati dinā . .

. . . . 2 adhe dāpaye dattaḥ	17 8* 2 3	adhenopari saṃ . . . uparima 41 verso.
------------------------------	--------------------	--

rāśi dvaya guṇaye	51 6* 2 3 ; . upari yukta kṛiyate eka-
-------------------	--------------------	--

pañchāśānām 51
6

sthāpanam | 1 53 . . . | phalam ā 17 tri . 2

J 3. Folio 41 is much damaged and the illustration (Plate xxviii) suggests a double leaf ; but the illustration is deceptive, for the cause of the uneven colour is the presence of gum on the original leaf. The find order is unknown : writing α4.
[41 recto.] This is undoubtedly closely connected with fol. 65 recto and the repair of fol. 41 and the separation of the two parts of fol. 65 would possibly make both intelligible.
[41 verso.] Not understood. *Possibly the 8 and 6 are change-ratios.

K

i. *udā°* || ko rāsi pañcha yutā *mūladaḥ* sā rāsis sapta hīna

59 recto,

mūlada ko so rāsir iti prashṇaḥ

$$\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline 0 & 5 & \text{yu}^\circ & \dots & \text{mū}^\circ & 0 & \text{sā} & 0 & 7+ & \text{mū}^\circ & 0 \\ \hline 1 & 1 & & & & 1 & & 1 & 1 & & 1 \\ \hline \end{array}$$

karanaṁ | †yuta hīnaṁ cha-m-ekatvaṁ† $\boxed{12}$ tad dalam $\boxed{6}$ dvi

hṛiṇaṁ $\boxed{4}$ dalam $\boxed{2}$ vargaṁ $\boxed{4}$ †hīne yutiṁ cha kartavyā† |

hīnaṁ $\boxed{7+}$ anena yuti $\boxed{11}$ eśa sā rāsi || asya pratyānayaṁ *kṛiyate*

$$\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline 11 & \text{yu}^\circ & 5 & \text{mū}^\circ & 4 & 11 & 7+ & \text{mū}^\circ & 2 \\ \hline 1 & & 1 & & 1 & 1 & 1 & & 1 \\ \hline \end{array}$$

pañchāśama sūtraṁ 50

ii. *sūtraṁ* | gavāṁ viśesha kartavyaṁ dhanam chaiva puna . .

.

K [59]. The find order is unknown but the *sūtra* number is 50 and it probably originally preceded fol. 60. The reverse is blank, which possibly means that there are portions of two leaves stuck together.

(i) *Example*.—What number with five added is a square and that same number with seven subtracted also being a square ? What is that number ? is the question.

Statement $x+5=s^2$, $x-7=t^2$.

Solution $\langle x = [\frac{1}{2}(\frac{5+7}{2}-2)]^2 + 7 = 11$ by steps thus > : having combined the added and subtracted numbers $5+7=12$; that halved $=6$; two subtracted 4 ; halved 2 ; squared 4 ; then the subtractive number (7) is to be added and by the addition of this $4+7=11$ and this is the required quantity.

Proof : $11+5=4^2$, $11-7=2^2$. See Part I, §81.

(ii) There appears to be a reference to this fragment on fol. 60 *recto* where *sūtra* 51 is closed.

L 1.

60 recto.

ekona-vimsatima | gāvo 10 | rūpa 8 | . . . | . . . vivaritāsti ||

gāvo	10	rūpa	8
	1		1

eka pañchāsama sūtram 51 ||

sūtram || āya vyaya viśesham tu vibhajya drishya saṁguṇam |
yal labdham sā bhavet kālam ayam prashne . ya vidhi ||

udā° || dvi-dine ārajaye pañcha tri-dine nava bhakshaye
bhāṇḍāgāram tasya trīṁsā kim kālam āra bhakshaṇam ||

di°	5	dināra	9	dri°
di°	2	dina	3	30

karaṇam | †āyā vyaya viśeshan tu† | tatrāyam | 5 |

5
2

[60.] Writing α2. Notice the 'sickle' i. Find order unknown. Connected with fol. 59 on one side and folios 61—63 on the other. Folios 60—63 form a fairly definite section (L) relating to earning and spending.

[60 recto.] (i) This fragment is connected with the sūtra at the bottom of fol. 59, but very vaguely.

(ii) Rule.—The known quantity is divided by the difference between the expenditure and earning. This result is the time

This means $t = \frac{s}{i-e}$

(iii) Example.—In two days one earns five ; in three days he consumes nine. His store is thirty. In what time will his earnings be consumed ?

Solution : $t = \frac{30}{\frac{5}{2} - 3} = 60$ and the amount earned in this time is $\frac{5}{2}$ of 60 = 150 dinaras.>

. bodi | phalam 180 | dvāpañchāsama sūtram 52 || 60 verso.

sūtram | aha dravya harāsauta tad viśesham vibhājayet
yal-labdham dviguṇam kālam° dattā sama-dhanā prati ||

[60 verso.] (i) Remnant of proof of the example on the obverse. The complete proof probably was :—

2 days : 5 dināra : : 60 days : 150 dināra

3 days : 9 dināra : : 60 days : 180 dināra

and 180—150=30.

(ii) Rule.—(If one earns e_1 in d_1 days and another e_2 in d_2 days and the first gives g to the second then $\frac{e_1}{d_1} t - g = \frac{e_2}{d_2} t + g$ and) $t =$

$$\frac{2g}{\frac{e_1}{d_1} - \frac{e_2}{d_2}}$$

L 1—contd.

iii. udā° | tri-dine ārjaye pañcha bhṛitako-m-eka pañḍitaḥ

dvitīyaṁ pañcha divase rasam ārjayate budhaḥ

prathamena dvitīyasya sapta dattā nidhānataḥ

datvā sama-dhanā jātā kena kālena katthyatām

5	rū	6
3	di	5

See *Indian Antiquary*, XLII (1888), pp. 41, 44 ; but in 1915 Dr. Hoernle sent me the following note :—" The textual difficulty was not fully understood by me : the text is badly corrupted ; a portion (the 2nd *pāda*) has dropped out, and another (the 1st *pāda*) has been mixed up with the commentary. The real text of the first *pāda* is quoted in obverse line 8 of the next folio, in the commentary of the second example of the *sūtra*, and the missing part of the second *pāda* must be supplied from obverse lb. 4 and 5 of *sūtra* 52 ; which is merely a variant of *sūtra* 53. The latter *sūtra* should really run as follows :—

ahadravya viśeṣaṁ cha vibhajya datta samguṇaṁ |
yal-labdham dviguṇam kṛtā dattā sama-dhanā prati ||

i.e., " the difference of the daily earnings, having divided (invested), is multiplied with the given amount : the result being doubled is the time ; the given amount goes towards making the possessions equal."

(iii) *Example*.—In three days one pandit earns a wage of five and a second wise man earns six (*rasa*) in five days. The second is given by the first seven from his store and by this giving their possessions become equal. Let it be stated in what time.

Solution : $t = \frac{2 \times 7}{5/3 - 1} = 30$.

L 2.

i. anena kālena sama-dhanā bhavanti ||

61 recto.

pratyayaṁ trai-rāśikena kṛiyate

3	5	30	pha°	50	prathame dvitīyasya (s) sapta dattā	7
1	1	1				
5	6	30		36	śeṣaṁ 43 43	
1	1					
			43		ete sama-dhanā jātā	

L 2. [61 recto.] i. The end of the solution of the example given on 60 verso.

Proof by the rule of three : 3 : 5 :: 30 : 50 and 5 : 6 :: 30 : 36 and 50—7=43=36+7.

L 2—contd.

ii. udā° || rājaputro dvayo kechi nṛipatis sevya santi vaiḥ
mekāsyāhne dvayash shaḍ bhāgā dvitīyasya divardhakam |
prathamena dvitīyasya daśa dīnāra dattavān
kena kālena samatām gaṇayitvā vadāśū me :||

13	3	dattam	10
6	2		1

karaṇam || †aha-dravya viśesham cha† | tatva

ii. *Example.*—Two Rājputs are the servants of a king. The wages of one are two and one-sixth a day, of the second one and one-half. The first gives to the second ten dīnāras. Calculate and tell me quickly in what time there will be equality. (Indian Antiquary, 1888, p. 44).

Statement: $\frac{13}{6}$, $\frac{3}{2}$, given 10.

Solution: The difference of the daily earnings,

Continued on the reverse.

i.	1	13	30	pha 65	prathamena dvitīyasya . 61 verso.
	1	6	1		
	1	3	30	pha 45 r dattā jātā
	1	2	1		

55 | 56 || sama dhanā jātā ||

ii. sūtram tri-pañchāsamaḥ sūtram 53 ||

sūtram || vikrayena krayam bhājyam rūpa hīnam punar bhajet

lābhena guṇaye tatra nīvi bhavati tatra cha :||

iii. udā° || dvibhi x kṛiṇāti yas sapta vikṛiṇāti tṛibhish shaḍ
ashtā-daśa bhaved lābhā kā nīvi tatra katthyatām :||

7	6	18	lābhā
2	3	1	

karaṇam | †vi

L 2. [61 verso.] i. Proof of example on the obverse—

1 : $\frac{13}{6}$: : 30 : 65

1 : $\frac{3}{2}$: : 30 : 45 and 65—10=45+10.

ii. The rule means $C = \frac{p}{c/s-1}$ where C is the capital, p the profit, c the rate of purchase and s the rate of sale.

iii. *Example.*—One buys 7 for 2 and sells 6 for 3 and 18 is his profit. What was his capital?

Solution.— $C = \frac{18}{\frac{1}{2}-\frac{1}{3}-1} = 24$. The proof is given on folio 62 recto.

L 3.

nīvi jātā | sya pratyaya *trairāśikena* ||

62 recto.

yadi dvibhis sapta labhyate | tadā chaturviṃśatibhi x kim |

2	7	24	phalaṁ rū° 84
1	1	1	

i. asya vikrayaṁ kṛiyate | yadi shadbhi traya . . labhyate tadā chaturāśitibhi x kim |

6	3	84	phalaṁ 42]	mūlaṁ 24		pātya śeṣhaṁ 18 eśa lābhāḥ
1	1	1					

chau-pañchāsama sūtraṁ 54.

ii. sūtraṁ || vikrayaṁ bhājaye chaiva guṇayet kraya piṇḍatām |

rūpone mūla guṇaye labdha lābhaṁ cha prāpyate ||

iii. udā° || dvibhi kṛiṇāti yas sapta vikṛiṇāti tribhish shat
mūlā chā

L 3. [62 recto.] i. Continued from folio 61 verso.

“If for two 7 are obtained, then what for twenty-four?”

2 : 7 :: 24 : 84 articles.

Again “If by six three are obtained then what for eighty-four?”

6 : 3 :: 84 : 42

and the original quantity was 24 and the difference 42—24=18.

ii. The rule means $p=C(c/s-1)$.

iii. Example.—Articles are bought at 7 for 2 and sold at 6 for 3.

i.

2	7	24	pha° 84
1	1	1	1

 atha vikrayaṁ

6	3	84
1	1	1

 62 verso.

pha° 42
1

 . . . 24 | pātya śeṣhaṁ 18 | eśa lābhaṁ ||

pañcha-pañchāsama sūtraṁ 55

ii. sūtraṁ || vikrayaṁ bhājaye chaiva guṇayet kraya piṇḍavāt

vibhaktam sa cha kartavyaṁ guṇaye miśrakam budhaḥ

yal labdham sā bhaven mūlaṁ yatch . chheshaṁ lābha piṇḍatām ||

L 3. [62 verso.] i. Solution.—Continued from the obverse; $p=24(\frac{2}{3}+\frac{3}{6}-1)=18$.

Proof.—2 : 7 :: 24 : 84 and 6 : 3 :: 84 : 42 and 42—24=18 is the profit.

ii. Rule.— $C=\frac{M}{c+s}$ where $M=C+p$ is called the ‘mixed’ quantity.

L 3—contd.

iii. udā° || tribhiś cha labhater asṭau chaturbhiś cha vikrayamśh shaṭ
 sa mūla lābham utpaṇṇa śataṁ śasṭi vimiśritam |
 kīṁ mūlaṁ kaścha lābham cha kathayed gaṇakottamaḥ ||

8	6	miśra 160
3	4	1

karaṇam | †vikrayam bhājaye chaiva guṇayet†

.

iii. *Example.*—Eight articles are obtained for three and six are sold for four. The sum of the capital and profit is one-hundred and sixty. State, O best of calculators, what was the capital and what is the profit.

The solution is lost except for the first quotation, but part of a proof is given on folio 63 recto. The solution was $\langle C = \frac{160}{1+\frac{1}{2}} = 90$ and the number of articles bought was $\frac{2}{3}$ of $90 = 240 \rangle$.

L 4.

i.

8	3	240	phalam 90
1	1	1	1

 63 recto.

6	4	240	phalam 160	mūlam	90	pātya śesham	70
1	1	1	1				

ii. shat pañchāśama sūtram 56

|| vikrayam cha vibhaktavyam guṇitam kraya rāsivat
 kṛitvā rūpa kshayam chaiva vibhaktam mūlam āpnuyāt

iii. udā° || pañchabhiś chatu vargam tu grihitam kena mānava
 . . kenash shat vikrītamśh shaṭ pañchaśa riṇam kṛitam |
 kraya vikraya saṁguṇya nīvis tasyaiva kathyatām ||

16	6	riṇam 56 +
5	1	1

. bhājaye chaiva

1
6

L 4. [63 recto.] i. Proof of example given on folio 62 verso.

8 : 3 :: 240 : 90 and 6 : 4 :: 240 : 160 and 160 = 90 + 70.

ii. The rule means $C = \frac{l}{1 - \frac{p}{s}}$ where l is the loss sustained, i.e., having investigated the selling rate multiply with the purchase rate and having subtracted from unity divide—and the capital is obtained.

iii. *Example.*—With five four-squared are obtained by some man. For one six are sold and fifty-six is the loss. Calculating purchase and sale let his capital be stated.

The solution is $\langle C = \frac{r l}{1 - \frac{p}{s}} = 120$ and the number of articles is $\frac{1}{5}$ of $120 = 364 \rangle$.

L 4—contd.

i. punāśya vikraya

6	1	384
1	1	1

 phalam

64

 mūlam

120

 63-verso.

chatush shashṭi pātya śesham

56

 eśa riṇam kṛi.

saptā-paṁchāśama sūtram 57.

ii. sūtram || vastra śulkaṁ yad bhavati tada . hṛita vastrataṁ |
traī-rāśika vidhānena śulka vikraya tatvataḥ ||

iii. udā° || paṭasya śulka viṁśāṁśam ka tris-śatam |
paṭa-kānām paṇa kṛite dvau patau hṛita śaulkikau |
. . . mūlyam paṇa daśas teshāḥ kiṁ mūlyam

L 4. [63 verso.] Proof of example on the obverse : $\langle \frac{1}{6} : 1 :: 384 : 120 \rangle$, then with the selling rate $6 : 1 : 384 : 64$ and $120 - 64 = 56$.
ii. Rule.—That which is the tax on cloth is taken in cloth : by the method of the rule of three tax and sale alike.
iii. The example is not understood but reads something like this : The tax on a piece of cloth is one-twentieth part. Some one sells three-hundred. On the pieces being brought to market, two pieces are taken by way of tax : ten is (?) the selling price. What is the value ?

M 1.

i.	1 20 rakti 1 1 4	dhā° 1 1 a° 0 ya° 4 1 4	su° 1 chhe° 80* rakti-su° rakti 1 1 1 1	pha° dha° 4 20 recto.
			ya° 1 pā° 3 mū 1	

puna tṛitīyasyaiva	2 20 1 1 4	1 3 1 1 chhe° 1* 2 2 1 chhe° 1* 4 4	1 1
--------------------	------------------	--	--------

. chhedam 6 dhā°-dra° pha° dhā° 4 ya° 1 pā° 2 mū° 1 ||
suvarṇasya māṇam samā

ii. udā° || sa pañcha nava bhāgāni dināni trayo-daśaḥ
. nām kim ||

M 1. [20 recto.] Section 'M' begins. Writing β.
i. A fragment of a solution or 'proof'. There were at least three statements, of which the second is $1\frac{1}{2}$ of 20 rakti : 1 dhā + $4\frac{1}{4}$ yā
:: 1 su° + 1 ra° : 4 dhā° + 1 yā° + 3 pā° + 1 mū° <or 25 ra° : 2025 mū° : : 81 ra° : 6561 mū°>.
Then a similar statement of the third (restored) $2\frac{1}{4}$ of 20 rakti : $\frac{1}{2}$ dra° + $1\frac{1}{2}$ dhā° + $\frac{1}{4}$ ya° : : 1 su° + 1 ra° : 4 dhā° + 1 yā° + 1 ka°
+ 2 pā° + 1 mū° <or 45 ra° : 3625 mū° : : 81 ra° : 6525 mū°>.
The numbers marked with asterisks are change-ratios. See Part I, §§ 103-104 ; and § 110 for the measures employed.
ii. Example.—Too mutilated to restore.

i. mū 12000 20 verso.

udāharaṇam | sarposhṭā-daśa hasto prāviśaty ārdhāṅgulaṁ
sa nava bhāga . . ti ekaviṁśati bhāgaṁ mapaharaṁti |
pratidīnenah kim kālena vīlaṁ saṁprāpyate ||

1 1 1 1 18 chhe° 24* an° ha°	phalaṁ va° 2 mā° 4 dī 10 $\frac{1}{2}$
2 21+ 1 360 1 1	
1	
9	

udāharaṇam | kīṇa x kilārdhaṅgulaṁ divase divase.

[20 verso.] i. A mere fragment : 12,000 mūdrikas
ii. Example.—A snake eighteen hastas long enters its hole at the rate of one-half plus one-ninth of that minus one-twenty-first part of an aṅgula a day. In what time will it have completely entered its hole ?
($\frac{1}{2} + \frac{1}{9} - \frac{1}{21}$) an° : $\frac{1}{210}$ years : : 18 × 24 an° : 2 years 4 months 10 $\frac{1}{2}$ days.
iii. Example.—A worm.....(see MAHAVIRA, V, 5).

M 2.

udā° || śumeru prithivi śamku surānām parimāśrayam ||

33 verso.

āga x kaśchi tarasā suramadiram ||

satatam sapta-sārdhānām sa pāmadhya . . . ||

sa tri-bhāgā tri-pañchāmśa nityam evam cha gachchhati |

yojanānām sahasrāṇichatur-āśītir uchchhṛitam |

kena kālena sau gachchhe vada me ta śuniśchitam ||

7	di° 1	yo° 84000	adha chchhedam 360* di
1	1	1	
2			

M 2. [33 verso.] *Example.*—From the home of the gods a certain person desires to ascend swiftly SUMERU, the pole of the Earth and the dwelling place of the gods. He goes constantly at the rate of seven times one and a half and its quarter with one-third and one-fifth. The height of Sumeru is eighty-four thousand *yojanas*. In what time will he reach the summit? Give me well considered answer.

There is some doubt about the rate of going and the only clear parts of the statement are the second and third terms (1 day and 84,000 *yojanas*), but possibly the complete statement was

$$7(1+\frac{1}{2})(1+\frac{1}{4})(\frac{1}{3}+\frac{1}{5})yo : 1 \text{ day} :: 84,000 yo : \frac{12000}{360} \text{ years} = 33\frac{1}{2} \text{ years.}$$

i. udā° || dīnāra ko nāma viśā . . ttī du x khārjanīyam sukha-bhojane cha | 33 recto.

tasyārdham ardham cha yad ardham ardham ta ke . deva guru prasādam

kṛipāṇa dhana bhuktaṁ ||

1	1	1	1	1	1	108	pha° dī° 1 dhā 8 d . .
1	2	2	2	2	2	1	

ii. uda° || ardham stāram nava roma śatāni cha |

dvādaśa stīti charmāni kati romā

	12	24			
	1	1			
900	12	24		12	24
	1	1		1	1

pha° roma

[33 recto.] i. *Example.*—The earning of *dīnāras* is difficult but consuming them is easy. One gives one-half increased by ration of one-half (six times) for food for the poor. What is the amount consumed in 108 days?

$$1 : \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} : 108 : 1 \text{ dī}^\circ 8 \text{ dhā}^\circ 1 \text{ am}^\circ$$

$$<\text{i.e., } \frac{1}{81} = 1\frac{1}{81} = 1 + \frac{1}{81} + \frac{1}{81} \text{ and } 4 \text{ am}^\circ = 1 \text{ dhā}^\circ \text{ and } 12 \text{ dhā}^\circ = 1 \text{ dī}^\circ >.$$

See Part I, § 110.

ii. *Example.*—(This is not understood, but appears to refer to the number of hairs on the skin of an animal.)

M 3.

i. . chandraanibhāṇa

32 recto.

. tu gaganam nita rāvaṇe | ra yam
 tyakta sutaya śetayā | sā kai kena parāvartam dhanur bhāga śa
 pa vane patamāṇasau daśa bhāgam nidhāryate | evam tat
 parimāṇa hīya mānam tu nityaśaḥ kiyatas tu parāvartaī bhūmim
 prāpyayate ja .

dha°	1	1	+	parā°	8	yoja°	30	chhe°	8000	yo°-ja°	°
	1	10					1		1		
	5										

phalam parā° 218181 śe

9
11

ii. udā° || nāga śva chchharma gāmi dratama daśa

M 3. [32 recto.] Folios 32 and 36 have the same knot.

i. A mutilated example about RĀVAṆA and (?) SITA. When Sitā had been carried up 30 *yojanas* into the air she dropped something to earth, which turned over 8 times in $1\frac{1}{10}$ *dhanus*. How many revolutions did it make before reaching the earth?*Solution.*—($1\frac{1}{10}$ — $\frac{1}{10}$) *dha*° : 8 revolutions : : 30 × 8,000 *dhanus* : : 218,181 $\frac{1}{11}$ revolutions. (There is a fair amount of conjecture here. See Part I, § 47).ii. *Example* (?).—A snake which is 100 *yojanas*, 6 *krośas*, 3 *hastas* and 5 *āṅgulas* long sheds its skin at the rate of 1 *āṅgula* in 2 days. In what time will it be free?

(The solution is given (?) on the reverse.)

i.

100	ūrdha chchhe° 768000 a°-yo°
6	1
8*	
3	
4000*	va° 429867 mā° 1 di° 4
5	
24*	

1
 ādha chchhedam 768000 phalam

32 verso.

ii. udā° || vraja . . chariśvāktā patitam bhūmi tale paṭam |

tri-śatāmsya . . nām tu sapta yojana hiyate |

chatur daśas tu koṭṭi . . hūyata pañcha-śasṭhi cha |

kai dinai bhūtale prāpya vada me ganakottama ||

nyāsa sthāponam . . kriyate |

[32 verso.] i. 1 *an*° : 2 d : : 100 *yo*° + 6 *kro*° + 3 *ha*° + 5 *an* : 429,867 years 1 month and 4 days.<or 1 : 2 : : 77,376,077 *an*° : $\frac{154752154}{360}$ years>. See Part I, § 108, for the measures employed.

ii. An example about some garment falling to the earth. The elements are uncertain. Compare with the problem on the obverse (i).

78946

M 4.

i. hyā pañcha triguṇita sakhe

36 recto.

. esha deśa pramāṇam samāptam ||

ii. udā || sa . . lavanasya rāshe koshṭhatām vā kṛitām rharai |

eshām chaikām rāśi punar e dhā nitā |

saptāṇām m api chaikā rāśis tulitāni |

pañcha saptatyā . . sahasram bhavet saptāshṭa guṇam kim

rā	1	1075	56
1	1	1	1

adha chchhedam 2000* pa°-bhā° | pha- bhā 30
pa° 200

esha rāśi lavaṇa pramāṇam

iii. kākinī daśa bhāgasya dadyād ashtādaśīti . . |

ṭasyām viṃśati bhāgas cha śata bhāgam prayachchhati |

naro vakshaśa

M 4. [36 recto.] i. 'This land measurement is completed' may refer to the fragmentary example at the bottom of folio 32 verso, but I doubt it.

ii. The example appears to refer to heaps of salt. If one heap or quantity weighs 1,075 *palas* how much will 56 heaps weigh?

1 : 1075 :: 56 : 30 bhā° + 200 pa°

< or $\frac{1075 \times 56}{2000} = 30 \frac{200}{1000}$ bhāra = 30 bhā° + 200 pa° >.

iii. One tenth of a cowry is given in eighty-eight.... Of this one-twentieth and one-hundredth.....

i.

ya°	3	1	1*
1	1	360	

yo°	5 chhe°	4608000*	ya°-yo°
1			

pha° va° 21333
mā° 4

36 verso.

ii. yojanasya tribhāgārdham sa tribhāga padonakam |

yā nau dinat tribhāgena . . . gena gachchhati |

śā puna φ pañcha bhāgārdham yojanasya tathāshṭamam

. ti nivartante vāyu vega valāhatā |

yojanānāmshṭau tara śatam kena kālena gachchhati ||

di	1	bhā	1	1	gu	1	1	1	3	bhā
	3		3	2		5	2+	8		

[36 verso.] i. The statement means 3 ya° : 1 day :: 5 yo° : 21,333 years 4 months or 3 ya° : $\frac{1}{360}$ years :: 5 × 4,608,000 ya° : 21,333 years 4 months < $\frac{5 \times 4,608,000}{5 \times 360} = 21,333\frac{1}{3}$ >. For the measures see Part I, § 108.

$\frac{109}{(\frac{1}{3} \text{ of } \frac{1}{3}) + 3 - (\frac{1}{3} \text{ of } \frac{1}{3}) + \frac{1}{3}} = < 1 \text{ year, 3 months, } 12\frac{2}{3} \text{ days} >.$

ii. A boat travels $\frac{1}{3}$ of $\frac{1}{3} + \frac{1}{3} - \frac{1}{3}$ yojanas in $\frac{1}{3}$ of a day, but is driven back by the wind $\frac{1}{3}$ of $\frac{1}{3}$ of a yojana in $\frac{1}{3}$ of 3 days. In what time will it travel 108 yojanas?

The problem is something like this but the details are not clear and the lower part of the statement has disappeared. See Part I, p. 51.

M 5.

i. *khagā ekādaśā* bhuktā prasṛitīm chaiva meva cha ||

34 recto.

. . shtau vada sakhe kiṁ khagaṁ vada sundari ||

pra° 1	kha° 11	khā 5760	phalaṁ khaga* 63360.
1	1	1	

esha bāhu pramāṇaṁ ||

ii. kaśchit pumāṁ suvaṇṇas tu kalā pāda yutaṁ yavaṁ ||

pratyaḥ sūline śuddhi kila dattavāṁ |

pamchābdai māśam evaṁ tu dinaī pañchadaśas tathāḥ

datvā . . sya sarvāya jñātum ichchhāmi tatvata ||

di 1	1	6* bhā	5	chchhedam 192* yava-tola
1	1	1		
	1	4		
	4*			

M 5.

[34 recto.] i. The problem is : Eleven birds feed on a *prasṛiti* (handful) of corn; how many can feed on 8 *Kharis* of corn? It ends "Say, O friend, say what are the *Khagas*, O SUNDARĪ."

If this is correct, the name Sundarī, 'beautiful one,' is used in exactly the same way as *Lilāvati* is used by Bhāskara.

The solution is 1 *pra*° : 11 *kha*° :: 8 *khā*° : 63,360 *khagas* which would make 720 *prasṛiti*=1 *khāri*; but there are many elements of doubt and the application of *esha bāhu pramāṇaṁ* to this particular problem is not clear.

ii. By certain persons one *kalā* plus one *pāda* and one *yava* are given in gold daily at the shrine of ŚULIN. What would be the amount of the gift in five years, five months and fifteen days.....I desire to know that.

Solution.—1 day : 1 *ya* + 1 *ka* + 1 *pa* :: 5 y. 5 m. 15 d. : x <or 1 d. : 30 *pa*° :: 1,965 d. : $\frac{1965 \times 30}{102 \times 25}$ *tola* = 12 *to*° + 3 *dhā*° + 1½ *am*° >

See Part I, § 111.

i.

34 verso.

chitṛitāṁgai | tānī yata śara-paramparay ārjunena griddhra tayā

spṛiṣānti	1	śa° 1	yoja° 777 1	8	phalaṁ . . . 940
	8	1	222 7	chchhe	
	1				
	9				
	1				
	5				

ii. māśakārdha yuto dhyanta vista pañchapañchāśa saterēṇa vajra maṇai labdham . . . tra kathayaśva mūlyam śāṇa chaturbhāgasya siddhārtha pañcha bhāgasya.

ku° 1	chhe° 128*	mā°-ku° 1	mā°	chhe 40*	si°-mā°	sa° 55
1	1	2				1
2						

[34 verso.] i. The fragment *chitṛitāṁgai* *ārjunena griddhra* is extremely interesting although it throws no light on the problem. See Part I, § 47.

The statement is puzzling : it may mean

$$\frac{1}{2} + \frac{1}{2.9} + \frac{1}{2.9.5} : 1 \text{ sa} :: 777 \text{ yo}^\circ + 222\frac{1}{2} \text{ kro}^\circ : \dots 40$$

But all the terms except the second are ambiguous.

ii. The problem is about a diamond weighing 1½ *māshaka*, and obtained for ? 55 *satera*.

The statement means 1½ *ku*° + ½ *mā*° : 55 *sa*°, and indicates that 128 *mā*° = 1 *ku*° and that 40 *si*° = 1 *mā*°. See Part I, § 111.

The whole page is an interesting puzzle. (Is the leaf a double one? Neither side shows any clear lenticle.)

M 6.

i. sūrya māṇasya

37 recto.

divākarasya ghaṭikaiḥ kim prayātasya vada . niśchitaṁ .

$$\left\| \begin{array}{c} 30 \text{ mu}^\circ \text{ chhe}^\circ 2^* \text{ gha}^\circ\text{-mu}^\circ \\ 1 \qquad \qquad \qquad 1 \qquad \qquad \qquad 1 \end{array} \right\| \left\| \begin{array}{c} 500,000,000 \\ 1 \end{array} \right\| \left\| \begin{array}{c} \text{gha}^\circ 1 \\ 1 \end{array} \right\| \text{pha}^\circ \text{yo}^\circ 83.333333\frac{1}{3}$$

ii. bhāṇo ratham sūra mahoraga siddhasaṁ(g)hai vidyādharaḥ φ parivritaṁ . . .

ahorātru | koṭi śatārdham sa ratham pryāsyāt tad brūhi śastra

kuśalo . . . vaktum || muhūrtam ekena kim gachchhe brūhi me

ganakottamā ||

$$\left\| \begin{array}{c} 500000000 \\ 1 \end{array} \right\| \left\| \begin{array}{c} \text{gha}^\circ 2 \\ 1 \end{array} \right\| \text{pha}^\circ \text{yo}^\circ 166,666,666\frac{2}{3}$$

M 6. [37 recto.] i. The question may be roughly restored : The Sun (sūrya) traverses 500,000,000 *yojanas* in a day. State with certainty the amount of the journey of the sun (Divākara) in a *ghaṭikā*.

The statement means $30 \text{ mu}^\circ : 500,000,000 : 1 \text{ gha}^\circ : 83,333,333\frac{1}{3} \text{ yo}^\circ$ and it indicates that $2 \text{ ghaṭika} = 1 \text{ muhūrta}$ ($= \frac{1}{30}$ of a day). The origin of the length of the daily journey of the sun, namely 500,000,000 *yojanas*, is not known. See Part I, § 100.

ii. The chariot of the sun (BHĀNU) is surrounded by the groups of gods, great snakes, SIDDHAS and VIDYĀDHARAS. In a day and night its journey is said to be half a hundred *koṭis*. Tell me, O best of calculators, how much in one *muhūrta* ?

$$30 \text{ mu}^\circ : 500,000,000 :: 2 \text{ gha}^\circ : 16,666,666\frac{2}{3} \text{ yo}^\circ.$$

i. bhāge bhaved rāśi |

37 verso.

ūrdha chhhedaṁ 108000 viliptāṇaṁ liptā 5

ii. pañchārdha saṁvatsare bhukte rāśaikā yadi bhānujaḥ brūhi . . . ka tatvajña

samaśve vāsareṇa kim

$$\left\| \begin{array}{c} 2 \\ 1 \\ 2 \end{array} \right\| \left\| \begin{array}{c} \text{rā}^\circ 1 \\ 1 \end{array} \right\| \left\| \begin{array}{c} 1 \text{ aṁ}^\circ \frac{1}{360} \\ 1 \end{array} \right\|$$

ūrdha chhhedaṁ 108000 viliptāṇaṁ rāśi | adha chchhedaṁ $\frac{1}{6}$ viliptā lipta ||

phalaṁ viliptā 2 || esha graha gatiṁ ||

iii. udā° || rāja yudhisthiro nāma φ pāṇḍu-vaṁśa

[57 verso.] i. The remnant of a problem possibly related to the daily motion of Jupiter, which according to the *Sūrya Siddhānta*, amounted to very nearly 5 minutes of arc (*liptā*).

ii. If BHĀNUJA (Saturn) moves through a sign in two and a half years, state, O knower of the truth, what will its motion in a solar day be equal to.

The solution is $2\frac{1}{2}$ years : 1 sign :: 1 degree : x

$$\text{and } x = \frac{1 \text{ sign} \times \frac{1}{2} \text{ degrees}}{2\frac{1}{2} \text{ years}} = \frac{30 \times 60 \times 60 \times 2}{5 \times 360} = \frac{108,000}{900} = 120'' = 2 \text{ minutes of arc (not 2 seconds as stated in the text, where } \text{riliptā}$$

appears to have been written by mistake for *liptā*). The terms employed are all orthodox except perhaps *vāsara* for 'solar day', but its special use is quite intelligible.

See Part I, § 100 ; and also my *Hindu Astronomy*, p. 57.

iii. This fragment is of interest because of the reference to YUDHISTHIRA. See Part I, § 48.

M 7.

i. vyūha pārtham hehayakī ghnata

47 verso,

sāyakaiś chaiva ϕ patti sva-pāda dala śodaśai |

a nyā chataśrā vai hatā tena mahātma vām ||

śarāṇām cha parimāṇam viśārada ||

śi	1	16	4 a° chhe°	21870	phalaṁ śarā .	2624400
	1	1	1	1		
		1				
		4				
		1				
		2				

anyā i pramāṇam

ii. sūtram || eko ratho gaja

M 7. [47 recto.] i. This appears to relate to PARTHA the Mahābhārata hero, who pierced each soldier with 16 $(1+\frac{1}{2})$ $(1+\frac{1}{2})$ arrows and slew four divisions of the army. How many arrows did he use?

1 śi°: 16 $(1+\frac{1}{2})$ $(1+\frac{1}{2})$:: $4 \times 21,870$: 2,624,400.

The abbreviation śi°=? ; a°=anikinī. See Part I, § 52.

There is a very similar example about Pārtha in the *Līlāvatī* (§ 67) which has already been quoted (Part I, § 47).

ii. Rule.—There is little doubt that this rule relates to the constitution of an army and is exemplified on the reverse (fol. 47 recto.)

i.

47 recto,

. vichakṣaṇaḥ

chamūs tu pritanās tisras tisraś cha

anikinī daśaguṇām āhu arakṣhohanī buddaḥ ||

[47 recto.] i. Apparently 3 chamūs=1 pritanā, 3 pritanās=1 anikinī and 10 anikinīs=1 akṣhauhiṇī. The statement mean: a patti consists of 1 ratha+1 gaja+5 nara+3 turaga (i.e., 1 chariot+1 elephant+5 foot soldiers+3 horsemen) and that an akṣhauhiṇī contains 3.710 of each of these, namely—

3.710.1	chariots	=21,870	chariots.
3.710.1	elephants	=21,870	elephants.
3.710.5	foot-men	=109,350	foot-men.
3.710.3	horsemen	=65,610	horsemen.

TOTAL =218,700.

Albirūnī (Chap. xlviii) gives the following scheme:—

Each akṣhauhiṇī has 10 anikinī.

anikinī	3	chamū.
chamū	3	pritanā.
pritanā	3	vāhinī.
vāhinī	3	gaṇa.
gaṇa	3	gulma.
gulma	3	senāmukha.
senāmukha	3	patti.
patti	1	ratha

and “a ratha comprehends besides, one elephant, three riders and five footmen.”

Possibly all these terms were included in the example but vāhinī, gaṇa, gulma and senāmukha are now missing. Numerically Albirūnī's scheme is identical with that given in our text.

The abbreviation tu° in the text is probably for turaga ‘a horse.’

See Part I, §§ 51 and 94.

M 7—contd.

akshohi

ra° 1	esha pati	3	3	3	3	3	3	10	gu°
ga° 1		1	1	1	1	1	1	1	
na° 5		gunitā jāta				ratha	21870		
tu° 3						gaja	21870		
						nara	109350		
						haya	65610		
							(218700)		

esha akshohiṇī pramāṇam || . .

ii. udā° || kaśchid rāja kumāra śātrudama |

ii. *Example.*—A certain prince SATRUDAMA [The phrase may as well mean: 'a certain prince (engaged in) curbing (his) enemies, (employed or fought so many soldiers)—K. N. D.]

M 8.

ki	di°	ra° 1	va° 3	chhe	48 recto
		ya° 1	1		
		3* bhā°	1		
		1	1		
		5	3		
		ka° 1	6* bhā°	mā° 3	
		1	1	1	
		4	12*		
		pā° 1	di° 1		
		4*	30*		
		śe° 1			
		3			

chhedam 480* rakti-pala gunitam jātam 41942 36 pala
115200

to° 8* pale-to° 3 tolen āsti dhā° 12* dhā° 7 dhāne nāsti am° 4* am° 2

M 8. [48 recto.] This is a statement belonging to some lost problem and, omitting the change-ratios (marked with asterisks), it means
5 days : 1 ra° + 1 ya° + 1 ka° + 1 pā + $\frac{1}{4}$ — $\frac{1}{8}$? : ? years + $\frac{1}{4}$ month + 1 day : 36 pa° + 3 to° + 7 dhā° + 2 am°
or 5 : 110 $\frac{1}{2}$ pā° — : ? years, etc. : 36 pa + 3 to + 7 dhā + 2 am°...
or 5 : $\frac{331}{115200}$ palas :: ? : $\frac{41942}{115200}$
(Therefore the third term must be of the order $\frac{5 \times 419420}{115200} \times \frac{115200}{330 \times 359}$ or nearly 180 years.) The abbreviations employed, the change-ratios, and the measures are explained in Part I, §§ 103 and 111.)

M 8—contd.

i.

...

phalam bhā° 2 enāsti 48 verso,

pala 2000 bhā° | pa° 270 || to° 8

chhe° 8*

dhā° 2

chhe° 12*

gum° 3

chhe° 5*

ya° 2

1

5

3* bhā

1

5

tola-pala to° 6 tole nāsti dhāne 12 dhā° 8 ||

ii. yadi dinam ekena esha dattam tad dvādaśa varsheṇa

di 1	216 bhā°	varshe 12 3 . . .	phalam bhāra 93 . . .
1	270 pa°	1 1	
	2000* chhe°		
	6 to°		
	8* chhe°		
	8 dhā°		
	12* chhe°		

M 8. [48 verso.] This exhibits two mutilated statements of proportion that evidently belong to the same problem.
 i. The first is ? : ? + 2 dhā° + 3 gum° + 2 ya° :: ? : 216 bhā° + 270 pa° + 6 to° + 8 dhā° +
 ii. If this is given in one day what is that in twelve years..... 1 day : 216 bhā° + 270 pa° + 6 to° + 8 dhā° + ... :: 12 years + ... : 93..... bhāra or <1 : 216½ bhāras (nearly) :: 4320 days (nearly) : x, and x = 216½ × 4320 = 933700 bhāra (roughly) >.

M 9.

. rakti kshaya pañcha guṇam 49 verso
 divasā vimśatikam kim śumdyati mah vada niśchayam

1	to° 3	kshaya 4+	va° 25	chhe° 360
1	mā° 2	60*	ma° 5	1
	12*	śi° 4	12*	
	am° 3	8*	di° 20	
	4*		30*	
	ya° 3			
	4*			
	ka° 1	6*		
	pā° 1	1		
	4*	4		
	mū° 1			
	4*			

62321	kshayam śodhya . . .	60881	adha chchhedam 2000 . . .
19200		19200	
sarva guṇitam	558278770		7 tola palam
	192·00		

M 9. [49 verso.] The statement means (omitting the change-ratios which are marked with asterisks) 1 day : 3 to° + 2 mā° + 3 am° + 3 ya° + 1 ka° + 1 pa° + 1 mū° — (+ 4 ra° + 4 śi°) :: 25 years + 5 months + 20 days : x
 or 1 day : $\frac{62321}{19200}$ tolas — $\frac{1440}{19200}$ tolas :: 9170 days : x, and x = $\frac{558,278,770}{10,200}$ tolas = bhā° + 1634 pa° + 5 to° + 0 mā° + 0 am° + 3 ya° + 3 ka° + 3 p° 1½ mū.

M 9—contd.

49 recto:

i. ya° 3 yavanāsti ka° 6

1 ka° 4 kalānāsti pā

4

. . pādanāsti mūdri° 4 pāmu mū° 2 ||

ii. udāharaṇam ||

. . . śūkhyair yajaṃti devī pratimahni kechit dadāmi devyā . . kaṃchaḥ

kṛtvā dīnāra śatāni chatvārīta dhānakā amḍikā raktikā yavā kalā pāda mūdrikā

cha | etad mūlyam vada me tatra m . . sya kim

1 to 12*	mū 400	dhā 1	phalaṃ dī 50 dīnāra nāsti dhāṇe
1 1	1	am 1	
		4*	12* dhānakā 10 dhāṇe nāsti am 4*
		ra 1	1* bhā
			1
			4
		ya 1	3* bhā
			1
			5
		ka 1	6* bhā
			1
			4
		pā 1	
		4*	
		mū 1	
		4*	
			am 1

[49 recto.] i. This is the end of the answer to the problem on 49 verso. See Part I, §§ 101 (iv) and 111.

ii. Example.—The first part is too broken up to make out, but it appears to refer to a gift connected with an image of Devī and worship by Śūkhyas. (cf. *Sūkhara*, the name of a Śaiva sect). [It is possible to read *Mūkhyair* for *Śūkhyair*, in which case the chiefs of some clan or territory are intended. K. N. D.]

The statement (omitting change-ratios) means—

$$1 \text{ to } : \text{cost } 400 :: 1 \text{ dhā}^\circ + 1 \text{ am}^\circ + 1 \text{ ra}^\circ + 1 \text{ ya}^\circ + 1 \text{ ka}^\circ + 1 \text{ pā}^\circ + 1 \text{ mū}^\circ : 50 \text{ dī}^\circ + 10 \text{ dhā}^\circ + 1 \text{ am}^\circ < \text{or } 12 \text{ dhā}^\circ : 400 \text{ dī}^\circ ::$$

$$\frac{2441}{1150} \text{ dhā}^\circ : x \text{ and } x = 50\frac{1}{2} \text{ dīnāras} >= 50 \text{ dī}^\circ + 10 \text{ dhā}^\circ + 1 \text{ am}^\circ.$$

M 10.

i.	to° 1 1	va° 5 1 3	to° 1 dhā° 1 12*	pha° va° 6 . . . še°	9 10	guṇitam 55 recto.
			am° 1 4*	7227 1200		
			ra° 1 1*	bhā°		
			ya° 1 3*	bhā°		
			si° 1 2*	bhā°		
			ka° 1 2*	bhā°		
			pā° 1 4*			
			mū° 1 4*			

atha śaḍḍrammako . . . jjarad, vidhānakais dramam śā . . . viṃśati-
pālā hatai dhānakā | asyaiva skandha-

ii.	to° 1 1	va° 5 1 3	to° 1 1	dhā° 1 12	1*	am° 1 1	1*	ra° 1 1	1*	ya° 1 1	1*
						48		60		192	
	si° 1 1	1*	ka° 1 1	1*	pā° 1 1	1*	mū° 1 1	1*			
		480		120		4800		19200			

M 10. [55 recto.] Folio 55 is here misplaced: it should come before folio 49, which has the same knot as 44.

i. The first statement means—

1 to° = 5½ years :: 1 to° + 1 dhā° + 1 am° + 1 ra° + 1 ya° + 1 si° + 1 ka° + 1 pā° + 1 mū° : x, and $x = 5\frac{1}{2} \times \frac{21681}{16200} = \frac{7227}{1200} = 6\frac{9}{100}$ years = 6 years, 8½ days. But the answer given appears to be 6 years

ii. This is the same proportion with the change-ratios given in cumulative form. See Part I, § 104.

. pañchatrīm satam |

55 verso.

divardha tolakasya divardha māśakasya .

divardha chāṇḍikā divardha yavasya kiṃ mūlyam ||

[55 verso.] If 1 tola cost thirty-five *drammas* what will be the price of one and a half *tolās*, one and a half *māśakas* and one and a half *āṇḍikas* and one and a half *yavas*.

M 10—contd.

nyāsa	to°	1	35	1	to°	pha° dram° 58 še°	31
		1	1	1			128
				2			
				1	1* mā°		
				1	6		
				2			
				1	1* am°		
				1	2		
				2			
				1	1* ya°		
				1	2		
				2			

punānyam

to°	1	35	1	1	1*	1	1*	1	1*	phalam 58 še°	31
	1	1	1	1	12	1	48	1	192		128
			2	2		2		2			

Statement.—(i) $1 \text{ to}^\circ : 35 :: 1\frac{1}{2} \text{ to}^\circ + 1\frac{1}{2} \text{ mā}^\circ + 1\frac{1}{2} \text{ am}^\circ + 1\frac{1}{2} \text{ ya}^\circ : 58\frac{31}{128} \text{ dram}^\circ$ or $<1 : 35 :: 319\frac{1}{2}/192 : 58\frac{31}{128}>$.

(ii) This is exactly the same proportion with cumulative change-ratios indicated. See Part I, §§ 104, 105.

M 11.

44 verso.

nīvī sapta-śatānām kaṣ kālām ārjana bhakshane ||

nyāsa sthāpanam kṛiyate

ā°	1	di°	1	bhā°	bha°	8	di°	5	bhā°	1	pa°	32	bhā	śū°	2	36	bhā°	1	1
	1		1			1		1		1		1			1	1	1	1	360
	2		3					3							2	4			

bhāṇḍā 700
1

vyaya rāśi	223	āya rāśi	280	etat kāleṇa ārjana bhaksh
	144		61	

M 11. [44 verso.] the capital is seven hundred. What is the time of the consumption of the earnings.

The statement means—

Daily earning $\frac{1}{144}$; given for Bha(vānī) 8 in $5\frac{1}{2}$ days; given for pa(ra-loka) 1 in 32; given for Śū(līn) $\frac{21}{4 \times 36}$; $\frac{1}{8}$ years; reserve 700.

<The daily earning is $\frac{1}{144}$ >. The expenditure quantity is $<\frac{8}{5\frac{1}{2}} + \frac{1}{32} + \frac{21}{4 \times 36} = > \frac{321}{144}$. <The daily loss is $\frac{321-162}{144} = \frac{61}{144}$ so 700 will last $\frac{700 \times 144}{61} = 162\frac{1}{61}$ years and 'in this time the earning will be consumed.'

Then 1 day : $\frac{321}{144} :: \frac{2}{3} \times 360 : 2559\frac{1}{3}$ and this is the (total) expenditure in $\frac{2}{3}$ years = 4 years, 7 months, $2\frac{2}{3}$ days.

Then the income, $1\frac{1}{2}$ days : $1\frac{1}{2} :: \frac{2}{3} \times 360 : 1859\frac{1}{3}$, and $2559\frac{1}{3} - 1859\frac{1}{3} = 700$.

M 11—contd.

di	1	223	280
	1	144	61

ūrdha chchhedam 360 phalam . . 2559 śe $\frac{1}{161}$ | esha

vyaye ||

. . .	va° 4 mā° 7 di° 2 śe°	28
		61

atha āya	di°	1	1	280
		1	1	61
		2	3	

i. . . .	2559	di	1	223	esha vyaya pramāṇam
	1		1	144	
	61				

44 recto.

ii. udā° || eka daśārdham utpati sa tribhāga dina dvayāt

pūjārtham sa tribhāgam cha trayodaśa . tatāś chayet

sāṣṭha bhāga dinā trīṇi vāsudevasya chārchayet

pādoṇa trayodaśāṇām cha aṣṭha sārḍha dināni chet ||

brāhmaṇā bhojane dadyā paraloka hitārthinaḥ

sa tribhāgam . jjarām sa pañcha bhāga dinattrayet

pa°

ardham sārḍham dine

[44 recto. 1. Again $\frac{280}{61} : 2559_{61} :: \frac{1}{360} : \frac{223}{144}$. This is the expenditure measure. See Part I, §96.

Example.—One produces ten and a half in two and one-third days. For the sake of religion he gives thirteen and one-third in three and one-eighth days; he offers for VĀSUDEVA one quarter less than thirteen in eight and a half days. Desiring reward in a future world he gave to Brāhmins for food one and one-third in three and one-fifth days two and a quarter in five days

M 12.

... .. ārayet
 . . sārdha dvādaśam evā tra bhojanē madyam uttamet
 sa tṛi bhāga trayastrimśai dinaid vāṇijyakasya tu. |
 bhāṇḍāre dvādaśa śata vajārāṇām sthitāsyā vai |
 eshā vyayasamutpattau kaḥ kālām brūhi paṇḍita ||
 karaṇa-vidhānena dvādaśa śatasya bhāṇḍāre stī ta .

10	2	bhā°	13	3	bhā°	13	8	bhā°	1	3	bhā°	1	1	bhā°	1	5	bhā°	2	1]	bhā°
1	1		1	1		1	+	1	1	1		2	1		1	1		[1	1	1
2	3		3	8		4	2		3	5		2			3			[4	4	4

12	33	bhā	1	1	bhāṇḍā	1200	gunītāni
1	1		1	360		1	
2	3						

M 12. [43 recto.] and also twelve and a half in thirty-three and one-third days for the best wine for the consumption of merchants. In the treasure house was stored twelve hundred. Say, O Pandit, how long can this expenditure continue.

The statement means :—

$$\text{Daily income} = \frac{10\frac{1}{2}}{2\frac{1}{2}} = 9.$$

$$\text{Daily expenditure} = \frac{13\frac{1}{2}}{3\frac{1}{2}} + \frac{13\frac{1}{2}}{6\frac{1}{2}} + \frac{1\frac{1}{2}}{3\frac{1}{2}} + \frac{1}{1\frac{1}{2}} + \frac{1}{6\frac{1}{2}} + \frac{2\frac{1}{2}}{5} + \frac{12\frac{1}{2}}{33\frac{1}{2}} = \frac{1807}{240}.$$

<The daily loss is therefore $\frac{1807}{240} - 9 = \frac{727}{240}$ and $\frac{1200}{727 \div 240} \times \frac{1}{1\frac{1}{2}} = \frac{800}{727}$ is the period>.

2	10	800	adha-chchhedam 360 diva . . teṇa saha	ya-piṇḍam	43 verso.
1	1	727			
3	2				

2982	adhunā vyaya piṇḍam	di°	1	1807	800
486		1		240	727
727					

ūrdha-chchhedam 360 phalam diva	2982	puna	800	2982	1
	486		727	486	1
	727			727	

adha-chchhedam 360 di . . phalam pratidina . . .	1807	evam sarva
	240	

trai-rāśikena | udā°

[43 verso.] *Proofs.*— $2\frac{1}{2} : 10\frac{1}{2} :: \frac{800}{727} \times 360 : 1782\frac{54}{7}$ the total amount earned and $1782\frac{54}{7} + 1200 = 2982\frac{54}{7}$.

Again $1 : \frac{1807}{240} :: \frac{800}{727} \times 360 : 2982\frac{54}{7}$; and lastly $\frac{800}{727} : 2982\frac{54}{7} :: \frac{1}{1\frac{1}{2}} : \frac{1807}{240}$ the daily expenditure. Thus each item (can be tested) by the rule of three.

M. 13.

ārdha yukte trayo-daśa sārdham bhavati

42 recto,

40 bhā°	160	13	eshām chchhedām kṛitā jātā ekeṇa . . .
1	1	1		
		2		

. . . sārdha trayo-daśabhi kim iti	1	4	27	pha° 54 eshām . .
	1	1	2	

. ekena labdha chatvārish śaḍbhi saṃpadyate katham	1	4
	1		

. . eko labhati chatvāri śansardhasya tu kim bhavet

.

M 13. [42 recto.] This contains portions of a solution that is not, at present, fully understood. The preliminary work is missing and then comes the following proportion $40 : 160 :: 13\frac{1}{2} : 54$, or cancelling by 40 we get $1 : 4 :: \frac{3}{2} : 54$. The next part is missing but apparently was—

$$1 : 4 :: 6 : 24$$

$$1 : 4 :: 3 : 12$$

$$1 : 4 :: \frac{3}{2} : 18$$

.

i. jātā 54 | śaḍbhi 24 | . . 12 | ardhā 18 | ekatraṃ 54 ||

42 verso,

e . . . trai-rāśika karaṇaḥ pratyeka mūlya vidhi ||

ii. aparaṃ vakshyāmi | vimśānām diva . . . kim prathamē khandhakeṣu yo

bhili khita apāsya prashnā vidhi	20	1	1	. . . guṇaye guṇitā
	1	1	3	
		2		

jātā	20	3	1	chhedam	20	1	1	bhāge . . . jātām phalam rū
	1	2	3		1	2		

10 || esha vimśānām diva . . bhavati | atra uparimāś khandhakasya esha guṇākāram bhavati |

[42 verso.] i. A fragment: $24 + 12 + (24 + 12) \div 2 = 54$ This sūtrā gives the three term solution with respect to one price.

ii. I shall instance another. what is that which is written in the first term? The solution is a matter of intelligence. $20 \times 1\frac{1}{2} \times \frac{1}{3} = 20 \times \frac{3}{2} \times \frac{1}{3} = 20 \times \frac{1}{2} \times 1 = 10$ Now this is the calculation of the foremost term.

M 14.

50 verso;

- i. *dramme trapusa śataṁ labdhaṁ ardheṇa labhyateḥ kati* |
eka rāśis tu kalanā gaṇita prakṛiyā kuruh

1 dramme	phalaṁ 50
100 trapusā	
1	
2	

- ii. *aparaṁ udā° || sārḍha dvaye . yasardha divardhe labhyateḥ kati* |

2
1
2
1
2
1
2
1
2

- iii. *sūtraṁ || ardhen opari saṁguṇya . . vardha krameṇa cha* |
ardheṇa ūrdhaṁ guṇaye ma . . pañcha saṁguṇe |
bhājaye labdha paṇyaṁ

M 14.

[50 verso.] i. The solution is 1 *dramma* : 100 *trapusā* :: $\frac{1}{2}$: 50.ii—iii. The problem is too mutilated to understand. The *sūtra* seems to apply to the problem, but it is not clear.

. vaśishta putra

50 recto.

sikasyārthe putra pautra upayogyaṁ bhavatuḥ

likhitaṁ Chchhajaka putra gaṇaka rāje brāhmaṇena |

sarveshām-m-eva śāstrāṇām gaṇitaṁ mūrdhni tiṣṭati |

ādyāvasāne saṁsāre utpaṁnna . . . mahat

paśchā śṛiṣṭi tadā kartuṁ śivena paramātmāna

. . yādyam cha-m-utpaṁnnaṁ gaṇitaṁ sakhya kāraṇam |

yach

[50 recto.] At the top of this page is the remnant of a problem, too broken up to make out. The rest of the page is devoted to what appears to be a colophon. This is not all clear but what remains seems to state that the work was written by a certain Brāhmaṇa, a prince of calculators, the son of Chhajaka. It also refers to the importance of the science of calculation, which, it is said, we owe to ŚIVA.

